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Composite Vectors and Scalars in Theories of Electroweak Symmetry Breaking

Antonio Enrique Cárcamo Hernández
antonio.carcamo@usms.cl

Advisor: Professor Riccardo Barbieri

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Abstract

In the context of a strongly coupled Electroweak Symmetry Breaking, composite triplet of heavy vectors belonging to the $SU(2)_{L+R}$ adjoint representation and a composite scalar singlet under $SU(2)_{L+R}$ may arise from a new strong interaction invariant under the global $SU(2)_L \times SU(2)_R$ symmetry, which is spontaneously broken down to $SU(2)_{L+R}$. This thesis consists of two parts. The first part is devoted to the study of the heavy composite vector pair production at the LHC via Vector Boson Fusion and Drell-Yan annihilation under the assumption that the interactions among these heavy vector states and with the Standard Model gauge bosons are described by a $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ Effective Chiral Lagrangian. The expected rates of multi-lepton events from the decay of the composite vectors are also given. The second part studies the associated production at the LHC of a composite vector with a composite scalar by Vector Boson Fusion and Drell-Yan annihilation in the framework of a $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ Effective Chiral Lagrangian with massive spin one fields and one singlet light scalar. The expected rates of same sign di-lepton and tri-lepton events from the decay of the composite vector and composite scalar final state are computed. The connection of the Effective Chiral Lagrangians with suitable gauge models is elucidated.

To my mother Yadira Esther, to my father Antonio Nicolás, to my sister Eliana Marcela, to my brother Juan David and to my girlfriend Emeline.

Contents

1 Introduction **3**

1.1 Statement of the problem and purpose of the thesis 3

1.2 The Electroweak Chiral Lagrangian 6

1.3 $W_L W_L \to W_L W_L$ and $W_L W_L \to f\bar{f}$ amplitudes 8

1.4 Adding a composite scalar 11

1.5 $W_L W_L \to W_L W_L$, $W_L W_L \to f\bar{f}$ and $W_L W_L \to hh$ amplitudes 12

2 Composite Vectors at the LHC **14**

2.1 Chiral Lagrangian with massive spin one fields 14

2.2 Longitudinal WW scattering amplitude 16

2.3 Unitarity condition on WW elastic scattering 18

2.4 $W_L W_L \to V_\lambda V_{\lambda'}$ helicity amplitudes 19

2.5 Asymptotic behaviour of the $W_L W_L \to V_\lambda V_{\lambda'}$ amplitudes 22

2.6 Drell–Yan production amplitudes 23

2.7 *Composite* versus *gauge* models 24

2.7.1 More than a single gauge vector 26

2.8 Pair production cross sections by vector boson fusion 28

2.9 Drell–Yan pair production cross sections 29

2.10 Same-sign di-lepton and tri-lepton events 30

3 A "composite" scalar-vector system at the LHC **32**

3.1 The basic Lagrangian 32

3.2 Two body $W_L W_L$ scattering amplitudes 34

3.3 Asymptotic amplitudes and parameter constraints 36

3.4 Associated production of Vh total cross sections 38

3.5 Same-sign di-lepton and tri-lepton events 41

4 Conclusions **44**

Acknowledgments **46**

References **47**

Appendix: A well behaved theory at all energies **51**

1 Introduction

1.1 Statement of the problem and purpose of the thesis

One of the most important issues to be settled by the LHC is whether the dynamics responsible for ElectroWeak Symmetry Breaking (EWSB) is weakly or strongly coupled. A weakly coupled dynamics describing the mechanism of the EWSB is provided by the Standard Model and its Supersymmetric extensions. In the Standard Model, the existence of one Higgs doublet is assumed in order to explain the generation of the masses of all the fermions and bosons. In addition to the 3 eaten up Goldstone bosons, the Higgs doublet contains one physical neutral scalar particle, called the Higgs boson, which is crucial for keeping under control unitarity in the elastic and inelastic channels of the gauge boson scattering and which allows us to extrapolate a weakly coupled model up to the Planck scale. A light Higgs boson can also successfully account for the ElectroWeak Precision Tests (EWPT).

The Higgs boson mass is the only unknown parameter in the symmetry breaking sector of the Standard Model. However, an upper bound on the mass of the Higgs boson can be set requiring that the quartic coupling in the Higgs self interaction potential, which grows with rising energy, should be finite at an energy scale Λ up to which the Standard Model is assumed to be valid. If the quartic coupling in the Higgs self interaction potential becomes large, which corresponds to a heavy Higgs boson, perturbation theory in terms of this coupling breaks down. In that case, the Higgs boson becomes strongly interacting. Moreover, the requirement of unitarity in longitudinal WW scattering can be used to set an upper bound on the mass of the Higgs boson. In the Standard Model, the scattering amplitude for longitudinal W bosons will violate unitarity when the mass of the Higgs boson takes values larger than about 1 TeV [1], which means that perturbation theory breaks down and the Standard Model becomes strongly interacting for a sufficiently heavy Higgs boson. The lower bound on the mass of the Higgs boson is determined from the requirement of vacuum stability of the scalar self interaction potential; this lower bound depends on the mass of the top quark and on the cutoff Λ up to which the Standard Model is assumed to be valid. The value of the aforementioned quartic coupling decreases when the top quark Yukawa coupling increases. For a cutoff $\Lambda = 10^{16}$ GeV corresponding to the Grand Unification scale, the requirement of vacuum stability of the scalar self interaction potential, implies a lower bound of about 130 GeV for the mass of the Higgs boson [2]. The Standard Model can be self consistent up to

very high energies provided that the Higgs boson is relatively light. For example, the consistency of the Standard Model up to the unification scale $\Lambda = 10^{16}$ GeV sets the Higgs boson mass in the range 130 GeV$\lesssim M_H \lesssim$ 180 GeV [2].

In spite of the very good agreement of the Standard Model predictions with experimental data, the Higgs boson is yet to be detected experimentally. Therefore one can say that the mechanism of EWSB responsible for the generation of the masses of all fermions and bosons remains to be explained. Moreover, the Standard Model has the hierarchy problem, which is the instability of the mass of the Higgs field against quantum corrections, which are proportional to the square of the cutoff. This means that in a quantum theory with a cutoff at the Planck scale $\Lambda \simeq 10^{19}$ GeV, the Higgs boson mass will have quantum corrections that will raise it to about the Planck scale unless an extreme fine-tuning of 34 decimals is performed in the bare squared mass. This is the naturalness problem of the Standard Model.

As there is no direct experimental evidence for a Higgs particle up to date, it is natural to ask what happens if we keep all the Standard Model fields, except the Higgs boson. One can for example think of a very heavy Higgs boson and build an effective field theory below the Higgs boson mass. The effective theory contains three of the four components of the Higgs doublet, which have become the longitudinal components of the $W^{\pm}$ and Z bosons, but not the fourth component $-$ the Higgs boson. This is the starting point of the Electroweak Chiral Lagrangian (EWCL) formulation, which is inspired by the Chiral Lagrangian approach to QCD at low energies and Chiral Perturbation Theory [3, 4, 5, 6, 7]. However, as is well known, the EWCL formulation does not pass the EWPT and the unitarity considerations for WW scattering (unitarity is violated at energies around 1.7 TeV).

These problems can perhaps be overcome if one considers EWSB mechanisms in the framework of a strongly interacting dynamics, where the theory becomes non-perturbative above the Fermi scale and the breaking is achieved through some condensate. In the strongly interacting picture of EWSB, many models have been proposed, which predict the existence of composite particles, e.g. composite scalars [8, 9, 10, 11, 12, 13], composite vector resonances [14, 15, 16, 17, 18, 19, 20], composite scalar and vector resonances [21, 22] and composite fermions [23]. The spin-0 and spin-1 resonances predicted by these models play a very important role in controlling unitarity in longi-

tudinal gauge boson scattering up to the cutoff $\Lambda \simeq 4\pi v$. For appropiate couplings and masses, the exchange of the composite resonances can perhaps also account for the EWPT. Furthermore, a composite scalar does not have the hierarchy problem since quantum corrections to its mass are saturated at the compositeness scale.

The phenomenology of heavy vector states at high-energy colliders [24, 25, 26], as well as their role in electroweak observables, is subject of intensive discussion. However, in most of the existing analyses specific dynamical assumptions are made such as considering these vector states as the gauge vectors of a spontaneously broken gauge symmetry. Recent studies [18, 27] show that these assumptions may be too restrictive for generic models based on strong dynamics at the TeV scale, and only going beyond these assumptions can one successfully account for the EWPT by solely considering exchange of heavy vectors. Altogether we find it potentially useful to take a model independent approach based on an effective Lagrangian description of the new particles coming from the strong dynamics with the incorporation of the relevant symmetries, whatever they are, exact or approximate. The composite spin-0, spin-1/2 or spin-1 states arising from the unknown strong interaction, which are bound states of more fundamental constituents held together by a new strong interaction, may be the lightest non standard particles and their discovery could provide the first clue of strong EWSB at the LHC.

To understand the underlying dynamics, several measurements and observations will certainly be required. It is assumed that this new strong dynamics supposedly breaking the Electroweak Symmetry is by itself invariant under a global $SU(2)_L \times SU(2)_R$ symmetry, which is spontaneously broken to the diagonal $SU(2)_{L+R}$ subgroup. After gauging the Standard Model gauge group, the $SU(2)_L \times SU(2)_R$ global symmetry of the new strong dynamics is broken down to the $SU(2)_{L+R}$ custodial group. It is also assumed that the strong dynamics responsible for the EWSB gives rise to composite triplet of heavy vectors degenerate in mass belonging to the adjoint representation of the custodial symmetry group. These heavy vector states have a mass below the cutoff $\Lambda \simeq 4\pi v$. The study of the heavy vector pair production is crucial for distinguishing the different models since it is sensitive to many couplings and in some sense more model dependent. The heavy vector pair production at the LHC by Vector Boson Fusion and Drell-Yan annihilation is studied in the first part of this thesis under the assumption that the interactions among these heavy vector states and with the Standard Model gauge bosons are described by a $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ Effective Chiral Lagrangian. The relevant parameter space is determined by minimizing the growing

energy behaviour of the scattering amplitudes for longitudinal Standard Model gauge bosons going into a pair of polarized vectors. The connection between a composite vector and a gauge vector of a spontaneously broken gauge symmetry is also investigated. The cross sections for vector pair production and the expected rates of multi-lepton events from the decay of such heavy vectors into Standard Model gauge bosons at the LHC have been computed.

In the second part of the thesis a light composite scalar, singlet under $SU(2)_{L+R}$ with mass $m_h \lesssim v$, is added to the $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ Effective Chiral Lagrangian. The interactions of this scalar with the Standard Model Gauge bosons and with the heavy vector pairs are introduced. The asymptotic behaviour of the elastic and inelastic channels of longitudinal SM gauge boson scattering is studied. The unitarity condition for the elastic channel of longitudinal SM gauge boson scattering is used to determine the relevant parameter space, in which the associated production of a heavy vector together with a scalar via Vector Boson Fusion and Drell-Yan annihilation at the LHC is studied. The total cross sections for the production at the LHC of a heavy vector in association with a scalar and the expected rates of same sign di-lepton and tri-lepton events from the decay of the composite vector and composite scalar final states are computed. A thorough phenomenological analysis and the evaluation of the backgrounds to such signals will be necessary to assess the visibility of composite vector pairs and composite vector-composite scalar final states at the LHC.

1.2 The Electroweak Chiral Lagrangian

Chiral Lagrangians have been extensively used to describe the phenomenon of spontaneous symmetry breaking in strong and in weak interactions. They can be regarded as the low energy limit of an underlying fundamental theory. The basis of this approach have been formulated by Weinberg to characterize the S matrix elements for pions interactions; after that Gasser and Leutwyler developed them building the Chiral Perturbation Theory, which describes low energy effects of strong interactions and was motivated by the fact that below the mass of the ρ meson, the Hadronic spectrum contains an octet of very light pseudoscalar particles (π, K, η) [28, 29]. Inspired by the Chiral Perturbation Theory Lagrangian formalism up to $O\left(p^4\right)$ developed by Ecker et al., used in the description of the low energy effects in QCD, the following EWCL can be used to formulate

the EWSB without the Higgs boson:

$$\mathcal{L}_{SB} = \frac{v^2}{4}\left\langle D_\mu U \left(D^\mu U\right)^\dagger\right\rangle - \frac{v}{\sqrt{2}}\sum_{i,j}\left(\bar{u}_L^{(i)} d_L^{(i)}\right) U \begin{pmatrix} \lambda_{ij}^u \, u_R^{(j)} \\ \lambda_{ij}^d \, d_R^{(j)} \end{pmatrix} + h.c \,, \tag{1.1}$$

where:

$$\begin{aligned} &U\left(x\right) = e^{i\hat{\pi}(x)/v}\,, \qquad \hat{\pi}\left(x\right) = \tau^a\pi^a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}\,, \\ &D_\mu U = \partial_\mu U - iB_\mu U + iUW_\mu\,, \qquad W_\mu = \tfrac{g}{2}\tau^a W_\mu^a\,, \qquad B_\mu = \tfrac{g'}{2}\tau^3 B_\mu^0\,, \end{aligned} \tag{1.2}$$

U is the matrix which contains the Goldstone boson fields π^a with $a = 1, 2, 3$, the τ^a are the ordinary Pauli matrices, $\langle\rangle$ denotes the trace over $SU(2)$, λ_{ij}^u and λ_{ij}^d are the up and down type quarks Yukawa couplings, respectively.

The transformation properties of the Goldstone fields under $SU(2)_L \times SU(2)_R$ are

$$u \equiv \sqrt{U} \to g_R u h^\dagger = h u g_L^\dagger\,, \tag{1.3}$$

where $h = h\left(u, g_L, g_R\right)$ is an element of $SU(2)_{L+R}$, as defined by this very equation [30]. The local $SU(2)_L \times U(1)_Y$ invariance is now manifest in the Lagrangian (1.1) with U transforming as

$$U \to g_L(x)\, U\, g_Y^\dagger(x)\,, \qquad g_L(x) = \exp\left(i\theta_L^a(x)\tau^a/2\right), \quad g_Y(x) = \exp\left(i\theta_Y(x)\tau^3/2\right). \tag{1.4}$$

and with the W, B and quark fields transforming in the usual way. The inclusion of the leptons is straightforward. In the unitary gauge $\langle U\rangle = 1$, it is immediate to see that the chiral Lagrangian (1.1) gives the mass terms for the W and Z gauge bosons with

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} = 1\,. \tag{1.5}$$

As is well known, this relation is the consequence of the larger approximate invariance of (1.1) under the $SU(2)_L \times SU(2)_R$ global transformations $U \to g_L\, U\, g_R^\dagger$, which is spontaneously broken to the diagonal custodial group $SU(2)_C = SU(2)_{L+R}$ by $\langle U\rangle = 1$, and explicitly broken by g' and $\lambda_{ij}^u \neq \lambda_{ij}^d$. In the limit $g' = 0$ and $\lambda_{ij}^u = \lambda_{ij}^d$, the $SU(2)_{L+R}$ custodial symmetry implies $M_W = M_Z$, which is replaced by eq.(1.5) at tree level for arbitrary g'. The pions transform as a triplet under the custodial symmetry group $SU(2)_{L+R}$, which plays the role of a weak isospin group when low energy pion interactions are considered.

A term like

$$c_3\, v^2 \, \left\langle T^3 U^\dagger D_\mu U \right\rangle^2 \tag{1.6}$$

invariant under the local $SU(2)_L \times U(1)_Y$ but not under the global $SU(2)_L \times SU(2)_R$ symmetry is therefore forbidden. Its presence would undo the $\rho = 1$ relation.

The effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{gauge} + \mathcal{L}_{SB}, \qquad \mathcal{L}_{gauge} = -\frac{1}{2g^2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle B_{\mu\nu}B^{\mu\nu}\rangle \tag{1.7}$$

provides an accurate description of particle physics, in some cases even beyond the tree level, at least up to energies below a cutoff [31]:

$$\Lambda = 4\pi v \approx 3 \text{ TeV} \tag{1.8}$$

when a loop expansion ceases to be meaningful. This Lagrangian is therefore meant to describe the spontaneous breaking of the electroweak local invariance $SU(2)_L \times U(1)_Y \to U(1)_Q$ by a strong dynamics which itself breaks a global symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to SU(2)_{L+R} \times U(1)_{B-L}$. It suffers, however, of two main problems [31]:

- The violation of unitarity in WW scattering, evaluated at the tree-level, below the cutoff Λ.
- The inconsistency of the electroweak observables S and T when compared with the experimental data if evaluated at the one-loop level with Λ as ultraviolet cutoff.

While the first problem requires that some action be taken, we shall not address in the following the second problem. The electroweak observables S and T will receive many contributions from different sources, among which cancellations may occur and which are difficult to control without an explicit model. Furthermore S and T will in general be sensitive to the physics at the cutoff, not controllable by the effective Lagrangians that we are using.

1.3 $W_LW_L \to W_LW_L$ and $W_LW_L \to f\bar{f}$ amplitudes

The bad high energy behaviour of the WW elastic scattering, as of the WW annihilation into a pair of fermions manifests itself when one considers longitudinally polarized vector bosons, W_L. In order to compute the $W_LW_L \to W_LW_L$ and $W_LW_L \to f\bar{f}$ amplitudes one takes into account

the Goldstone Equivalence Theorem which states that at high energies the amplitude for the emission or absorption of longitudinally-polarized vector boson becomes equal to the amplitude for the emission or absorption of Goldstone field π [31]. In particular the $W_L^aW_L^b \to W_L^cW_L^d$ and $W_L^aW_L^b \to f\bar{f}$ scattering amplitudes at high energies become equal to the $\pi^a\pi^b \to \pi^c\pi^d$ and $\pi^a\pi^b \to f\bar{f}$ scattering amplitudes up to corrections of the order $O\left(\frac{M_W^2}{\sqrt{s}}\right)$ (and up to a factor of i^N where N is the number of Goldstone bosons):

$$A(W_L^aW_L^b \to W_L^cW_L^d) = A\left(\pi^a\pi^b \to \pi^c\pi^d\right)\left[1+O\left(\frac{M_W}{\sqrt{s}}\right)\right], \tag{1.9}$$

$$A(W_L^aW_L^b \to f\bar{f}) = -A\left(\pi^a\pi^b \to f\bar{f}\right)\left[1+O\left(\frac{M_W}{\sqrt{s}}\right)\right]. \tag{1.10}$$

Taking the $g' \to 0$ limit for simplicity, isospin conservation implies that the four pions Lorentz invariant scattering amplitude can be written as:

$$A\left(\pi^a\pi^b \to \pi^c\pi^d\right) = A\left(s,t,u\right)^{\pi\pi\to\pi\pi}\delta^{ab}\delta^{cd} + B^{\pi\pi\to\pi\pi}\left(s,t,u\right)\delta^{ac}\delta^{bd} + C\left(s,t,u\right)^{\pi\pi\to\pi\pi}\delta^{ad}\delta^{bc}. \tag{1.11}$$

The Bose symmetry implies that the four pions scattering amplitude should be invariant under the exchange of pions, that is, under the exchange $a \leftrightarrow b$, $t \leftrightarrow u$ and $a \leftrightarrow c$, $s \leftrightarrow t$. Then, the following relations are obtained:

$$B\left(s,t,u\right)^{\pi\pi\to\pi\pi} = A\left(t,s,u\right)^{\pi\pi\to\pi\pi}, \qquad C\left(s,t,u\right)^{\pi\pi\to\pi\pi} = A\left(u,t,s\right)^{\pi\pi\to\pi\pi}, \tag{1.12}$$

which implies that the four pions scattering amplitude has the following form:

$$A\left(\pi^a\pi^b \to \pi^c\pi^d\right) = A\left(s,t,u\right)^{\pi\pi\to\pi\pi}\delta^{ab}\delta^{cd} + A\left(t,s,u\right)^{\pi\pi\to\pi\pi}\delta^{ac}\delta^{bd} + A\left(u,t,s\right)^{\pi\pi\to\pi\pi}\delta^{ad}\delta^{bc}. \tag{1.13}$$

The function $A\left(s,t,u\right)^{\pi\pi\to\pi\pi}$ comes from the derivative interaction

$$\mathcal{L}^{\pi^4} = \frac{1}{48v^2}\left\langle\left[\pi,\partial_\mu\pi\right]\left[\pi,\partial^\mu\pi\right]\right\rangle = -\frac{1}{6v^2}\varepsilon^{abe}\varepsilon^{cde}\pi^a\pi^c\partial_\mu\pi^b\partial^\mu\pi^d \tag{1.14}$$

among the four Goldstones contained in the kinetic term of U in (1.1) and is given by:

$$A\left(s,t,u\right)^{\pi\pi\to\pi\pi} = \frac{s}{v^2}. \tag{1.15}$$

The growth of the $W_L^aW_L^b \to W_L^cW_L^d$ scattering amplitude with the square of the center of mass

energy $\sqrt{s}$ implies a violation of perturbative unitarity.

To determine the energy at which the perturbative unitarity is violated, the WW scattering amplitude is decomposed into partial waves and the unitarity condition in the $I = 0$ isospin channel is applied. The fixed isospin amplitudes are given by [18]:

$$T\left(I=0\right) = 3A\left(s,t,u\right) + A\left(t,s,u\right) + A\left(u,t,s\right) = 2A\left(s,t,u\right), \tag{1.16}$$

$$T\left(I=1\right) = A\left(t,s,u\right) - A\left(u,t,s\right), \tag{1.17}$$

$$T\left(I=2\right) = A\left(t,s,u\right) + A\left(u,t,s\right) = -A\left(s,t,u\right) \tag{1.18}$$

and the partial wave coefficients have the following form:

$$a_l^I\left(s\right) = \frac{1}{64\pi}\int_{-1}^{1} d\left(\cos\theta\right) P_l\left(\cos\theta\right) T\left(I\right). \tag{1.19}$$

Then, it follows that the partial wave coefficient $a_0^0\left(s\right)$ of isospin zero for the four pion scattering is given by:

$$a_0^0\left(s\right) = \frac{1}{32\pi}\int_{-1}^{1} dy A\left(s,t\left(y\right),u\left(y\right)\right) = \frac{s}{16\pi v^2}. \tag{1.20}$$

The strongest unitarity constraint $|a_0^0\left(s\right)| < 1$ implies:

$$\sqrt{s} < 1.7\,TeV. \tag{1.21}$$

This means that perturbative unitarity in WW scattering is violated at energies $\sqrt{s} \approx 1.7$ TeV, implying that New Physics should manifest itself at energies in the TeV range to restore unitarity in the scattering amplitudes of longitudinal gauge bosons.

From $SU(2)_{L+R}$ invariance and Bose symmetry, the $\pi^a\pi^b \to f\bar{f}$ scattering amplitude is given by:

$$A(\pi^a\pi^b \to f\bar{f}) = A(s,t,u)^{\pi\pi\to f\bar{f}}\delta^{ab} \tag{1.22}$$

where the leading contribution to this amplitude comes from the $\pi^2 f\bar{f}$ contact interaction also contained in (1.1) so that the function $A(s,t,u)^{\pi\pi\to f\bar{f}}$ is given by:

$$A(s,t,u)^{\pi\pi\to f\bar{f}} = \frac{m_f\sqrt{s}}{v^2}, \tag{1.23}$$

m_f being the fermion mass. In this case, one has that the $W_L^a W_L^b \to f\bar{f}$ scattering amplitude has an asymptotic behaviour which goes as $\frac{m_f\sqrt{s}}{v^2}$ at high energies.

The fact that the $W_L W_L \to W_L W_L$ and $W_L W_L \to f\bar{f}$ scattering amplitudes grow at high energies as $\frac{s}{v^2}$ and $\frac{m_f\sqrt{s}}{v^2}$, respectively implies the following two possibilities:

- New particles should exist in order to restore unitarity well before perturbativity is lost. In this case we have a weakly coupled EWSB, possibly extrapolable to much higher energies than $4\pi v$.
- The $W_L W_L \to W_L W_L$ and $W_L W_L \to f\bar{f}$ scattering amplitudes grow strongly until the interaction among the four W's and between two W's and fermion-antifermion pair becomes non-perturbative. Nevertheless, somewhat before this to happen, some new degrees of freedom produced by the strong dynamics may emerge at the TeV scale. The ultraviolet behaviour of the $W_L W_L \to W_L W_L$ and $W_L W_L \to f\bar{f}$ scattering amplitudes may be softened by the exchange of such massive composite states. In this case the appearance of new composite degrees of freedom from a strong sector could be the earliest manifestation of a strongly coupled EWSB.

It is worth to mention that the chiral formulation has the merit of isolating the problem to the sector of the Lagrangian which leads to the mass terms for the vector bosons and the fermions. Regardless of the type of dynamics ruling the EWSB mechanism, an ultraviolet completion of the EWCL given in (1.1) will have to exist. The key assumption here is that the EWCL catches the main physics below the cutoff, including the properties of the new composite particles lighter than the cutoff itself.

1.4 Adding a composite scalar

The simplest extension of the minimal EWCL is to add a new scalar field $h(x)$ singlet under $SU(2)_L \times SU(2)_R$. Since an elementary scalar has the hierarchy problem, a composite scalar arising from an unspecified strong dynamics is introduced so that quantum corrections to its mass are saturated at the compositeness scale. It is assumed that the Standard Model Gauge bosons are coupled to the strong sector via weak gauging: the operators involving the field strengths $W_{\mu\nu}$ and $B_{\mu\nu}$ will appear with loop suppressed coefficients, so that they can be neglected [32]. Another

assumption that is made is that the Standard Model fermions are coupled to the strong sector only via the (proto)-Yukawa interactions.

Under these assumptions the most general EWSB Lagrangian has three free parameters a, b and c [1] at the quadratic order in h and is given by [32]:

$$
\begin{aligned}
\mathcal{L}_{EWSB} =& \frac{1}{2}\left(\partial_\mu h\right)^2 - V(h) + \frac{v^2}{4}\left\langle D_\mu U \left(D^\mu U\right)^\dagger \right\rangle \left(1 + 2a\,\frac{h}{v} + b\,\frac{h^2}{v^2}\right) \\
& - \frac{v}{\sqrt{2}} \sum_{i,j} \left(\bar{u}_L^{(i)} d_L^{(i)}\right) U \left(1 + c\,\frac{h}{v}\right) \begin{pmatrix} \lambda_{ij}^u\, u_R^{(j)} \\ \lambda_{ij}^d\, d_R^{(j)} \end{pmatrix} + h.c
\end{aligned}
\tag{1.24}
$$

where $V(h)$ is some potential, including a mass term, for h. As we shall see, each of these parameters controls the unitarization of a different sector of the theory.

1.5 $W_L W_L \to W_L W_L$, $W_L W_L \to f\bar{f}$ and $W_L W_L \to hh$ amplitudes

As before, the $W_L^a W_L^b \to hh$ scattering amplitude at high energies such that $\sqrt{s} >> M_W^2$ is given by:

$$
A(W_L^a W_L^b \to hh) = -A\left(\pi^a \pi^b \to hh\right)\left[1 + O\left(\frac{M_W}{\sqrt{s}}\right)\right]. \tag{1.25}
$$

The function $A\left(s,t,u\right)^{\pi\pi\to\pi\pi}$ receives contributions from the four pion contact interaction π^4 and from the scalar exchange h and is given by:

$$
A\left(s,t,u\right)^{\pi\pi\to\pi\pi} = \left(1 - a^2\right)\frac{s}{v^2} + \frac{a^2 m_h^2 s}{v^2\left(s - m_h^2\right)} \tag{1.26}
$$

so that the strength of the four pion scattering amplitude is controlled by the parameter a. For $a = 1$ the exchange of the scalar unitarizes the four pions scattering amplitude and then the $W_L W_L \to W_L W_L$ scattering amplitude at high energies. In the case in which $a \neq 1$, one has a strong $W_L W_L \to W_L W_L$ scattering with violation of perturbative unitarity at energies $\sqrt{s} \approx 4\pi v/\sqrt{1-a^2}$.

[1] In general c will be a matrix in flavor space, but in the following it is assumed for simplicitly that it is proportional to unity in the basis in which the mass matrix is diagonal. This guarantees the absence of flavour changing neutral effects originated from the tree level exchange of h.

The leading contributions to the amplitude $A\left(\pi^a\pi^b \to f\bar{f}\right)$ come from the $\pi^2 f\bar{f}$ contact interaction and from the scalar exchange so that the function $A(s,t,u)^{\pi\pi\to f\bar{f}}$ is given by:

$$A(s,t,u)^{\pi\pi\to f\bar{f}} = \frac{m_f\,(1-ac)\sqrt{s}}{v^2}. \tag{1.27}$$

Then the parameters a and c control the strength of the $W_L W_L \to f\bar{f}$ scattering amplitude. Perturbative unitarity is satisfied for $ac = 1$.

On the other hand, the $\pi^a\pi^b \to hh$ scattering amplitude $\mathcal{A}\left(\pi^a\pi^b \to hh\right)$ receives contributions from the $\pi^2 h^2$ contact interaction and from the π and h exchanges, so that the function $\mathcal{A}\,(s,t,u)^{\pi\pi\to hh}$ is given by:

$$\mathcal{A}\,(s,t,u)^{\pi\pi\to hh} = -\frac{1}{v^2}\left(s\left(b-a^2\right) + \frac{3asm_h^2}{2\left(s-m_h^2\right)} - 2a^2m_h^2 + \frac{a^2m_h^4}{t} + \frac{a^2m_h^4}{u}\right). \tag{1.28}$$

This amplitude will not grow with the center of mass energy, that is, the perturbative unitarity condition is satisfied only for $b = a^2$. Hence, taking all conditions at the same time, only for the choice $a = b = c = 1$ the EWSB sector is weakly interacting (provided that the scalar h is sufficiently light). It is not surprising that $a = b = c = 1$ precisely corresponds to the Standard Model case with $h(x)$ being part, together with the π's, of a linear Higgs doublet. Away from the unitarity point $a = b = c = 1$, the scalar exchange alone will fail to fully unitarize the amplitudes for the elastic and inelastic channels of WW scattering. In this case the theory will become strongly interacting at high energies. Since the Goldstone Equivalence Theorem implies that the longitudinal polarization states of W and Z play the role of the pions in the new strong interaction, any collider process involving the W and Z bosons in the initial and final states can be helpful for an experimental study of the new strong interaction. In particular discovering a Higgs-like boson and at the same time finding an excess of events in $WW \to WW$ scattering at the LHC when compared with the prediction of the Standard Model will be a signal of the growing energy behaviour of the $WW \to WW$ scattering amplitude and then an experimental manifestation of strong EWSB. Besides that, the observation of the $WW \to hh$ scattering at the LHC, which in the Standard Model has an extremely small cross section might provide an experimental evidence of composite Higgs model and strong EWSB. The advantange of the $WW \to hh$ channel with respect to the $WW \to WW$ elastic channel comes from the fact that the first is the only process providing information on the parameter b and does not have pollution from transverse modes of the W [31].

2 Composite Vectors at the LHC

2.1 Chiral Lagrangian with massive spin one fields

In this chapter we shall consider the addition to the minimal EWCL of spin-1 states, triplet under $SU(2)_{L+R}$ in analogy with the ρ-states of QCD. This will allows us to study the interactions of these vectors, V_μ^a, with the W and Z in a comprehensive way. Especially in low-energy QCD studies, the heavy spin-1 states are often described by antisymmetric tensors [28, 29]. Here we shall on the contrary make use of the more conventional Lorentz vectors, belonging to the adjoint representation of $SU(2)_{L+R}$,

$$V_\mu = \frac{1}{\sqrt{2}}\tau^a V_\mu^a \,, \qquad\qquad V^\mu \to h V^\mu h^\dagger, \tag{2.1}$$

with h defined in (1.3).

The $SU(2)_L \times SU(2)_R$-invariant kinetic Lagrangian for the heavy spin-1 fields is given by

$$\mathcal{L}^V_{\rm kin} = -\frac{1}{4}\left\langle \hat{V}^{\mu\nu}\hat{V}_{\mu\nu}\right\rangle + \frac{M_V^2}{2}\left\langle V^\mu V_\mu\right\rangle \,. \tag{2.2}$$

Here $\hat{V}_{\mu\nu} = \nabla_\mu V_\nu - \nabla_\nu V_\mu$ and

$$\nabla_\mu V_\nu = \partial_\mu V_\nu + [\Gamma_\mu, V_\nu]\,, \qquad \Gamma_\mu = \frac{1}{2}\Big[u^\dagger\left(\partial_\mu - iB_\mu\right)u + u\left(\partial_\mu - iW_\mu\right)u^\dagger\Big]\,, \qquad \Gamma_\mu^\dagger = -\Gamma_\mu, \tag{2.3}$$

where u is defined in (1.3). Note that the covariant derivative given in the previous expression transforms homogeneously as V_μ itself does. The other quantity that transforms covariantly is $u_\mu = u_\mu^\dagger = iu^\dagger D_\mu U u^\dagger$, which, under $SU(2)_{L+R}$, has the following transformation rule: $u_\mu \to h u_\mu h^\dagger$.

In terms of these quantities the most general invariant terms up to a given number of vector indices is easily constructed. Assuming parity invariance of the new strong interaction, the full set of interactions up to cubic terms in the spin-1 fields is:

$$\mathcal{L}^V_{\rm int} = \mathcal{L}_{\rm 1V} + \mathcal{L}_{\rm 2V} + \mathcal{L}_{\rm 3V}\,, \tag{2.4}$$

where $\mathcal{L}_{1V}$, $\mathcal{L}_{2V}$ and $\mathcal{L}_{3V}$ are given by:

$$\mathcal{L}_{1V} = -\frac{ig_V}{2\sqrt{2}}\left\langle \hat{V}_{\mu\nu}[u^\mu, u^\nu]\right\rangle - \frac{f_V}{2\sqrt{2}}\left\langle \hat{V}_{\mu\nu}(uW^{\mu\nu}u^\dagger + u^\dagger B^{\mu\nu}u)\right\rangle, \tag{2.5}$$

$$\begin{aligned}\mathcal{L}_{2V} =& g_1 \langle V_\mu V^\mu u^\alpha u_\alpha\rangle + g_2 \langle V_\mu u^\alpha V^\mu u_\alpha\rangle + g_3 \langle V_\mu V_\nu [u^\mu, u^\nu]\rangle + g_4 \langle V_\mu V_\nu \{u^\mu, u^\nu\}\rangle \\ &+ g_5 \langle V_\mu (u^\mu V_\nu u^\nu + u^\nu V_\nu u^\mu)\rangle + ig_6 \left\langle V_\mu V_\nu (uW^{\mu\nu}u^\dagger + u^\dagger B^{\mu\nu}u)\right\rangle ,\end{aligned} \tag{2.6}$$

$$\mathcal{L}_{3V} = \frac{ig_K}{2\sqrt{2}}\left\langle \hat{V}_{\mu\nu}V^\mu V^\nu\right\rangle . \tag{2.7}$$

Every parameter in (2.4) is dimensionless. The interactions of a single vector with the EW gauge bosons are described by $\mathcal{L}_{1V}$ and have been extensively considered in the literature. The interactions which modify the W^2V^2 and WV^2 vertex are described by $\mathcal{L}_{2V}$ and have not been considered in the literature as well as the vector self-interactions given by $\mathcal{L}_{3V}$. It can be seen that the interactions of two pions (two longitudinal weak bosons) with the vector field V_μ are characterized by a coupling g_V. The interactions of the vector field V_μ with one longitudinal and one transverse gauge boson are characterized by the couplings f_V and g_V. Another important fact is the mixing of the vector field V_μ with the Standard Model Gauge fields; this mixing is proportional to gf_V.

In (2.4) we are not including:

- Operators involving 4 V's, since they are not relevant to the amplitudes considered in this work.
- Operators of dimension higher than 4, which we assume to be weighted by inverse powers of the cutoff $\Lambda \approx 3$ TeV, as suggested by naive dimensional analysis. As such, they would contribute to the VV-production amplitudes at c.o.m. energies sufficiently below Λ by small terms relative to the ones that we are going to compute.
- Direct couplings between any fermion of the SM and the composite vectors. This is plausible if the SM fermions are *elementary*. The third generation doublet could be an exception here. If this were the case, with a large enough coupling, this would not change any of the VV-production amplitudes, but might lead to a dominant decay mode of the composite vectors into top and/or bottom quarks, rather than into W, Z pairs.

As we are going to see, for a consistent description of high energy WW scattering we also have to add 4-derivative terms only involving the π-fields. Their most general form is:

$$\mathcal{L}_{contact} = c_1 \langle [u^\mu, u^\nu] [u_\mu, u_\nu] \rangle + c_2 \langle \{u^\mu, u^\nu\} \{u_\mu, u_\nu\} \rangle , \tag{2.8}$$

so that the total lagrangian will be:

$$\mathcal{L}^V = \mathcal{L}_\chi + \mathcal{L}^V_{\text{kin}} + \mathcal{L}^V_{\text{int}} + \mathcal{L}_{\text{contact}}. \tag{2.9}$$

2.2 Longitudinal WW scattering amplitude

For the process $\pi^a \pi^b \to \pi^c \pi^d$ in the center of mass frame we have:

$$p^\mu_a = (E, 0, 0, p) = \left(\sqrt{\frac{s}{4}}, 0, 0, \sqrt{\frac{s}{4}} \right), \qquad p^\mu_b = (E, 0, 0, -p) = \left(\sqrt{\frac{s}{4}}, 0, 0, -\sqrt{\frac{s}{4}} \right), \tag{2.10}$$

$$p^\mu_c = (E, k \sin\theta_{CM}, 0, k\cos\theta_{CM}) = \left(\sqrt{\frac{s}{4}}, \sqrt{\frac{s}{4}} \sin\theta_{CM}, 0, \sqrt{\frac{s}{4}} \cos\theta_{CM} \right), \tag{2.11}$$

$$p^\mu_d = (E, -k \sin\theta_{CM}, 0, -k\cos\theta_{CM}) = \left(\sqrt{\frac{s}{4}}, -\sqrt{\frac{s}{4}} \sin\theta_{CM}, 0, -\sqrt{\frac{s}{4}} \cos\theta_{CM} \right), \tag{2.12}$$

where θ_{CM} is the scattering angle in the center of mass frame, with:

$$t = -\frac{s}{2} (1 - \cos\theta_{CM}), \qquad u = -\frac{s}{2} (1 + \cos\theta_{CM}), \tag{2.13}$$

and the Madelstam variables are given by:

$$\begin{aligned} s &= \left(p^a + p^b\right)^2 = \left(p^c + p^d\right)^2 = E^2_{CM}, \qquad t = (p^a - p^c)^2 = \left(p^b - p^d\right)^2, \\ u &= \left(p^a - p^d\right)^2 = \left(p^b - p^c\right)^2, \end{aligned} \tag{2.14}$$

where E_{CM} is the center of mass energy.

The contribution due to the four point contact interaction contained in (1.14) to the four pions scattering amplitude is:

$$A \left(\pi^a \pi^b \to \pi^c \pi^d\right)_{\pi^4} = \frac{s}{v^2} \delta^{ab} \delta^{cd} + \frac{t}{v^2} \delta^{ac} \delta^{bd} + \frac{u}{v^2} \delta^{ad} \delta^{bc}. \tag{2.15}$$

On the other hand, expanding the contact interaction in $\mathcal{L}_{\text{contact}}$ to 4-th order in the π-fields, we obtain:

$$\mathcal{L}_1^{\pi^4} = -\frac{8c_1}{v^4}\varepsilon^{abe}\varepsilon^{cde}g^{\mu\lambda}g^{\nu\rho}\partial_\mu\pi^a\partial_\nu\pi^b\partial_\lambda\pi^c\partial_\rho\pi^d, \tag{2.16}$$

$$\mathcal{L}_2^{\pi^4} = \frac{8c_2}{v^4}\delta^{ab}\delta^{cd}g^{\mu\lambda}g^{\nu\rho}\partial_\mu\pi^a\partial_\nu\pi^b\partial_\lambda\pi^c\partial_\rho\pi^d. \tag{2.17}$$

From the previous expressions, the following contributions due to $\mathcal{L}_1^{\pi^4}$ and $\mathcal{L}_2^{\pi^4}$ to the function $A(s,t,u)$ of the expression (1.13) are obtained:

$$A_{\pi^4}^{(1)}(s,t,u) = -\frac{8c_1}{v^4}\left(s^2+2ut\right), \qquad A_{\pi^4}^{(2)}(s,t,u) = \frac{8c_2}{v^4}\left(t^2+u^2\right). \tag{2.18}$$

The Lagrangian which describes the $\pi^2 V$ interaction is given by

$$\mathcal{L}^{\pi^2 V} = \frac{g_V}{v^2}\varepsilon^{abe}\left(g^{\mu\kappa}g^{\nu\eta}-g^{\mu\eta}g^{\nu\kappa}\right)\partial_\mu\pi^a\partial_\nu\pi^b\partial_\kappa V_\eta^e, \tag{2.19}$$

which implies that the contribution due to $\mathcal{L}^{\pi^2 V}$ to the function $A(s,t,u)$ of the expression (1.13) is:

$$A_V(s,t,u) = \frac{g_V^2}{v^4}\left[s^2+2ut+M_V^2\left(\frac{t(u-s)}{t-M_V^2}-\frac{u(s-t)}{u-M_V^2}\right)\right]. \tag{2.20}$$

Therefore, the function $A(s,t,u)$ which describes the four pions scattering amplitude is given by:

$$A(s,t,u) = \frac{s}{v^2}-\frac{8c_1}{v^4}\left(s^2+2ut\right)+\frac{8c_2}{v^4}\left(t^2+u^2\right)+\frac{g_V^2}{v^4}\left[s^2+2ut+M_V^2\left(\frac{t(u-s)}{t-M_V^2}-\frac{u(s-t)}{u-M_V^2}\right)\right]. \tag{2.21}$$

The cancellation of the terms which go as $\frac{s^2}{v^4}$ in the four pions scattering amplitude is guaranteed only when:

$$c_2 = 0, \qquad c_1 = \frac{g_V^2}{8}, \tag{2.22}$$

which we shall adopt from now on. We shall come back to these relations in the following.

In this case, the function $A(s,t,u)$ takes the following form:

$$A(s,t,u) = \frac{s}{v^2}-\frac{G_V^2}{v^4}\left[3s+M_V^2\left(\frac{s-u}{t-M_V^2}+\frac{s-t}{u-M_V^2}\right)\right] \tag{2.23}$$

where we have set $g_V M_V = G_V$.

2.3 Unitarity condition on WW elastic scattering

Imposing the condition

$$G_V = \frac{v}{\sqrt{3}} \tag{2.24}$$

gives a good high energy behaviour of the WW scattering amplitude. This may be however a too strong condition. We shall be content by requiring no violation of unitarity for $\sqrt{s}$ below the cutoff Λ.

The partial wave coefficient $a_0^0(s)$ of isospin zero for the four pion scattering is given by:

$$a_0^0(s) = \frac{1}{32\pi}\int_{-1}^{1} dy A(s, t(y), u(y)) = \frac{M_V^2}{16\pi v^2}\left\{x\left(1 - \frac{3G_V^2}{v^2}\right) + \frac{2G_V^2}{v^2}\left[\left(2 + x^{-1}\right)\ln(x+1) - 1\right]\right\} \tag{2.25}$$

where

$$x = \frac{s}{M_V^2}, \qquad y = \cos\theta_{CM}. \tag{2.26}$$

The strongest unitarity constraint $|a_0^0(s)| < 1$ for any energy up to $\sqrt{s} = \Lambda$ implies:

$$\left|a_0^0\left(s = \Lambda^2\right)\right| = \frac{M_V^2}{16\pi v^2}\left|\left\{\frac{\Lambda^2}{M_V^2}\left(1 - \frac{3G_V^2}{v^2}\right) + \frac{2G_V^2}{v^2}\left[\left(2 + \frac{M_V^2}{\Lambda^2}\right)\ln\left(\frac{\Lambda^2}{M_V^2} + 1\right) - 1\right]\right\}\right| < 1. \tag{2.27}$$

Imposing the strongest unitarity constraint up to $\Lambda = 4\pi v \simeq 3$ TeV, the allowed region in the (M_V, G_V) plane is obtained and shown in Figure 1.

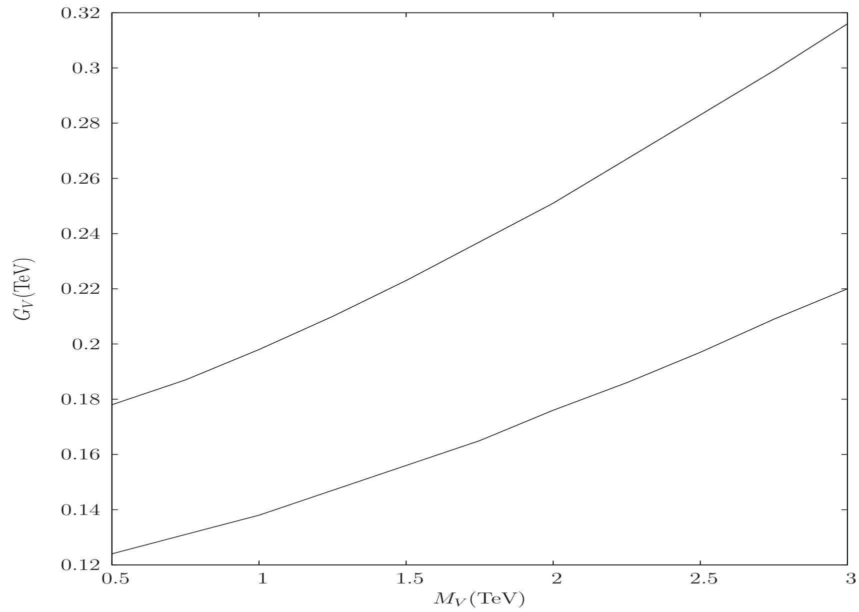


Figure 1: Strongest unitarity constraint in the (M_V, G_V) plane for the process $\pi^a\pi^b \to \pi^c\pi^d$ at $\sqrt{s} = 3$ TeV.

2.4 $W_L W_L \to V_\lambda V_{\lambda'}$ helicity amplitudes

In this Section we calculate the scattering amplitudes for two longitudinal W-bosons into a pair of heavy vectors of any helicity $\lambda, \lambda' = L, +, -$. To simplify the explicit formulae, we take full advantage of $SU(2)_{L+R}$ invariance by considering the $g' = 0$ limit, so that $Z \approx W^3$. We also work at high energy, such that

$$\sqrt{s},\ \sqrt{-t},\ \sqrt{-u},\ M_V >> M_W\,, \tag{2.28}$$

which allows us to make use of the equivalence theorem, i.e.

$$\mathcal{A}(W_L^a W_L^b \to V_\lambda^c V_{\lambda'}^d) \approx -\mathcal{A}(\pi^a\pi^b \to V_\lambda^c V_{\lambda'}^d)\,. \tag{2.29}$$

This restriction will be dropped in Sections 2.8 and 2.9, where we shall present numerical results, although the limitations of the effective Lagrangian approach will remain.

There are in fact four such independent amplitudes:

$$\mathcal{A}(W_L^a W_L^b \to V_L^c V_L^d)\,, \tag{2.30}$$

$$\mathcal{A}(W_L^a W_L^b \to V_+^c V_-^d)\,, \tag{2.31}$$

$$\mathcal{A}(W_L^a W_L^b \to V_+^c V_+^d) = \mathcal{A}(W_L^a W_L^b \to V_-^c V_-^d) \tag{2.32}$$

and

$$\mathcal{A}(W_L^a W_L^b \to V_L^c V_+^d) = -\mathcal{A}(W_L^a W_L^b \to V_L^c V_-^d)\,. \tag{2.33}$$

By $SU(2)_{L+R}$ invariance the general form of these amplitudes is

$$\mathcal{A}(W_L^a W_L^b \to V_\lambda^c V_{\lambda'}^d) = \mathcal{A}_{\lambda\lambda'}(s,t,u)\delta^{ab}\delta^{cd} + \mathcal{B}_{\lambda\lambda'}(s,t,u)\delta^{ac}\delta^{bd} + \mathcal{C}_{\lambda\lambda'}(s,t,u)\delta^{ad}\delta^{bc}\,, \tag{2.34}$$

where, by Bose symmetry, it is simple to prove that

$$\mathcal{A}_{\lambda\lambda'}(s,t,u) = \mathcal{A}_{\lambda\lambda'}(s,u,t) \text{ and } \mathcal{C}_{\lambda\lambda'}(s,t,u) = \mathcal{B}_{\lambda\lambda'}(s,u,t) \text{ for } \lambda\lambda' = LL, +-, ++\,, \tag{2.35}$$

whereas

$$\mathcal{A}_{L+}(s,t,u) = -\mathcal{A}_{L+}(s,u,t) \text{ and } \mathcal{C}_{L+}(s,t,u) = -\mathcal{B}_{L+}(s,u,t)\,. \tag{2.36}$$

These amplitudes receive contributions from:

i) contact interactions, $\pi^2 V^2$, contained in $\mathcal{L}_{\rm kin}^V$ and proportional to unity (with an overall $1/v^2$ factored out) or contained in $\mathcal{L}_{2V}$ and proportional to $g_i, i = 1, \ldots, 5$;

ii) one-π exchange, proportional to g_V^2, contained in $\mathcal{L}_{1V}$;

iii) one-V exchange, proportional to $g_V g_K$, with g_V contained in $\mathcal{L}_{1V}$ and g_K in $\mathcal{L}_{3V}$.

For ease of the reading, we keep first only the contributions with $\mathcal{L}_{2V}$ and $\mathcal{L}_{3V}$ set to zero, so that[2]:

- For $\lambda\lambda' = LL$

$$\mathcal{A}_{LL}^{1V} = -\frac{G_V^2 s}{v^4\,(s - 4M_V^2)}\left[\frac{\left(t + M_V^2\right)^2}{t} + \frac{\left(u + M_V^2\right)^2}{u}\right]\,, \tag{2.37}$$

$$\mathcal{B}_{LL}^{1V} = \frac{u-t}{2v^2} + \frac{G_V^2 s\,(u + M_V^2)^2}{v^4 u\,(s - 4M_V^2)}\,. \tag{2.38}$$

[2] In all these functions the variables are in the order (s,t,u) and are left understood.

- For $\lambda\lambda' = +-$

$$\mathcal{A}^{1V}_{+-} = \frac{2G_V^2 M_V^2 \,(t+u)\,(tu - M_V^4)}{v^4 tu\,(s-4M_V^2)}\,, \tag{2.39}$$

$$\mathcal{B}^{1V}_{+-} = \frac{2G_V^2 M_V^2 \,(M_V^4 - tu)}{uv^4\,(s-4M_V^2)}\,. \tag{2.40}$$

- For $\lambda\lambda' = ++$

$$\mathcal{A}^{1V}_{++} = \frac{2G_V^2 M_V^2 \,(t+u)\,(M_V^4 - tu)}{v^4 tu\,(s-4M_V^2)}\,, \tag{2.41}$$

$$\mathcal{B}^{1V}_{++} = \frac{(t-u)}{2v^2} - \frac{2G_V^2 M_V^2 \,(M_V^4 - tu)}{uv^4\,(s-4M_V^2)}\,. \tag{2.42}$$

- For $\lambda\lambda' = L+$

$$\mathcal{A}^{1V}_{L+} = \frac{\sqrt{2}G_V^2 M_V^3 \,(t-u)\,\sqrt{s\,(tu - M_V^4)}}{v^4 tu\,(s-4M_V^2)}\,, \tag{2.43}$$

$$\mathcal{B}^{1V}_{L+} = -\frac{\sqrt{s\,(tu-M_V^4)}\,\{v^2 su + 4M_V^2\,[G_V^2\,(M_V^2+u) - v^2 u]\}}{2\sqrt{2}uv^4 M_V\,(s-4M_V^2)}\,. \tag{2.44}$$

Here and in the following, we set

$$G_V \equiv g_V M_V\,,\;\; F_V \equiv f_V M_V\,, \tag{2.45}$$

adopting a notation familiar in the description of spin-1 states by anti-symmetric Lorenz tensor fields.

Switching on $\mathcal{L}_{2V}$ and $\mathcal{L}_{3V}$ gives an extra contribution to the various amplitudes:

- For $\lambda\lambda' = LL$

$$\Delta\mathcal{A}_{LL} = (g_1 - g_2)\,\frac{s\,(s-2M_V^2)}{v^2 M_V^2} + (g_4 - g_5)\,\frac{s\,[2M_V^2\,(3M_V^2 - s) + t^2 + u^2]}{v^2 M_V^2\,(s-4M_V^2)}\,, \tag{2.46}$$

$$\Delta\mathcal{B}_{LL} = g_2 \frac{s\,(s-2M_V^2)}{v^2 M_V^2} + \frac{s\,(t-u)}{v^2 M_V^2}\left(g_3 + \frac{g_K g_V}{4}\,\frac{s+2M_V^2}{s-M_V^2}\right) + g_5 \frac{s\,[2M_V^2\,(3M_V^2-s)+t^2+u^2]}{v^2 M_V^2\,(s-4M_V^2)}. \tag{2.47}$$

- For $\lambda\lambda' = +-$

$$\Delta\mathcal{A}_{+-} = 4\,(g_4 - g_5)\,\frac{(M_V^4 - tu)}{v^2\,(s-4M_V^2)}\,, \tag{2.48}$$

$$\Delta\mathcal{B}_{+-} = 4g_5 \frac{(M_V^4 - tu)}{v^2\,(s-4M_V^2)}\,. \tag{2.49}$$

- For $\lambda\lambda' = ++$

$$\Delta\mathcal{A}_{++} = 2\,(g_1 - g_2)\,\frac{s}{v^2} + 4\,(g_4 - g_5)\,\frac{(tu - M_V^4)}{v^2\,(s - 4M_V^2)}\,, \tag{2.50}$$

$$\Delta\mathcal{B}_{++} = 2g_2\frac{s}{v^2} + \frac{4g_5\,(tu - M_V^4)}{v^2\,(s - 4M_V^2)} - \frac{g_K g_V s(t-u)}{2v^2\,(s - M_V^2)}\,. \tag{2.51}$$

- For $\lambda\lambda' = L+$

$$\Delta\mathcal{A}_{L+} = (g_4 - g_5)\,\frac{(t-u)\,\sqrt{2s\,(tu - M_V^4)}}{v^2 M_V\,(s - 4M_V^2)}\,, \tag{2.52}$$

$$\Delta\mathcal{B}_{L+} = \frac{\sqrt{2s\,(tu - M_V^4)}}{v^2 M_V}\left[g_5\frac{t-u}{s - 4M_V^2} + \left(g_3 + \frac{g_K g_V}{2}\frac{s}{s - M_V^2}\right)\right]\,. \tag{2.53}$$

2.5 Asymptotic behaviour of the $W_L W_L \to V_\lambda V_{\lambda'}$ amplitudes

For arbitrary values of the parameters all these amplitudes grow at least as s/v^2 and some as $s^2/(v^2 M_V^2)$ or as $s^{3/2}/(v^2 M_V)$. As readily seen from these equations, there is on the other hand a unique choice of the various parameters that makes all these amplitudes growing at most like s/v^2, i.e.

$$g_V g_K = 1, \quad g_3 = -\frac{1}{4}, \quad g_1 = g_2 = g_4 = g_5 = 0, \tag{2.54}$$

whereas f_V and g_6 are irrelevant. With this choice of parameters the various helicity amplitudes simplify to

- For $\lambda\lambda' = LL$

$$\mathcal{A}_{LL}^{\text{gauge}} = -\frac{G_V^2 s}{v^4\,(s - 4M_V^2)}\left[\frac{(t + M_V^2)^2}{t} + \frac{(u + M_V^2)^2}{u}\right]\,, \tag{2.55}$$

$$\mathcal{B}_{LL}^{\text{gauge}} = \frac{u-t}{2v^2} + \frac{G_V^2 s\,(u + M_V^2)^2}{v^4 u\,(s - 4M_V^2)} - \frac{3s(u-t)}{4v^2\,(s - M_V^2)}\,. \tag{2.56}$$

- For $\lambda\lambda' = +-$

$$\mathcal{A}_{+-}^{\text{gauge}} = \frac{2G_V^2 M_V^2\,(t+u)\,(tu - M_V^4)}{v^4 tu\,(s - 4M_V^2)}\,, \tag{2.57}$$

$$\mathcal{B}_{+-}^{\text{gauge}} = \frac{2G_V^2 M_V^2\,(M_V^4 - tu)}{uv^4\,(s - 4M_V^2)}\,. \tag{2.58}$$

- For $\lambda\lambda' = ++$

$$\mathcal{A}_{++}^{\rm gauge} = \frac{2G_V^2 M_V^2\,(t+u)\,(M_V^4 - tu)}{v^4 tu\,(s - 4M_V^2)}\,, \tag{2.59}$$

$$\mathcal{B}_{++}^{\rm gauge} = -\frac{M_V^2\,(t-u)}{2v^2\,(s - M_V^2)} - \frac{2G_V^2 M_V^2\,(M_V^4 - tu)}{uv^4\,(s - 4M_V^2)}\,. \tag{2.60}$$

- For $\lambda\lambda' = L+$

$$\mathcal{A}_{L+}^{\rm gauge} = \frac{\sqrt{2}G_V^2 M_V^3\,(t-u)\,\sqrt{s\,(tu - M_V^4)}}{v^4 tu\,(s - 4M_V^2)}\,, \tag{2.61}$$

$$\mathcal{B}_{L+}^{\rm gauge} = -\frac{\sqrt{2}G_V^2 M_V\,(M_V^2 + u)\,\sqrt{s\,(tu - M_V^4)}}{uv^4\,(s - 4M_V^2)} + \frac{M_V\sqrt{s\,(tu - M_V^4)}}{\sqrt{2}v^2\,(s - M_V^2)}\,. \tag{2.62}$$

We show in Section 2.7 that the relations (2.54), and so the special form of the $W_L W_L \to V_\lambda V_{\lambda'}$ helicity amplitudes, arise in a minimal gauge model for the vector V_μ. In the generic framework considered here, some deviations from (2.54) may occur. In such a case the asymptotic behaviour of the various amplitudes will have to be improved, e.g., by the occurrence of heavier composite states, vectors and/or scalars, with appropriate couplings. Note in any event that, even sticking to the relations (2.54), the amplitudes for longitudinally-polarized vectors grow as s/v^2 for any value of G_V^2.

2.6 Drell–Yan production amplitudes

At the parton level there are four Drell–Yan production amplitudes, related to each other by $SU(2)$- invariance (in the g' limit, as usual):

$$|\mathcal{A}(u\bar{d} \to V^+V^0)| = |\mathcal{A}(d\bar{u} \to V^-V^0)| = \sqrt{2}|\mathcal{A}(u\bar{u} \to V^+V^-)| = \sqrt{2}|\mathcal{A}(d\bar{d} \to V^+V^-)|. \tag{2.63}$$

They receive contributions from: i) $W(Z)$-exchange diagrams, with the $W(Z)$ coupled to a pair of composite vectors either through their covariant kinetic term, $\mathcal{L}_{\rm kin}^V$, or via g_6 in $\mathcal{L}_{2V}$; ii) light-heavy vector mixing diagrams proportional to $f_V g_K$ with these couplings contained in $\mathcal{L}_{1V}$ and $\mathcal{L}_{3V}$. Their modulus squared, summed over the polarizations of the final-state vectors and averaged over colour and polarization of the initial fermions, can be written as

$$< |\mathcal{A}(u\bar{d} \to V^+V^0)|^2 >= \frac{g^4}{1536 M_V^6 s^2 (s - M_V^2)^2} F(s, t-u, M_V^2), \tag{2.64}$$

with F organized in different powers of s:

$$F(s,t-u,M_V^2) = F^{(6)}(s,t-u,M_V^2) + F^{(5)}(s,t-u,M_V^2) + F^{(\leq 4)}(s,t-u,M_V^2)\,, \tag{2.65}$$

where

$$F^{(6)} = (g_K f_V - 4g_6)^2 M_V^2 s^4 [s^2 - (t-u)^2], \tag{2.66}$$

$$\begin{aligned} F^{(5)} &= 4M_V^4 s^3 \left\{ (g_K f_V - 4g_6)^2 \left[2s^2 + (t-u)^2\right] \right. \\ &\left. + (g_K f_V - 4g_6) \left[2\,(7g_6 - 3)\, s^2 + 2\,(g_6 - 1)\,(t-u)^2\right] + 2\,(1-2g_6)^2 \left[s^2 + (t-u)^2\right] \right\}, \end{aligned} \tag{2.67}$$

$$\begin{aligned} F^{(\leq 4)} &= 4M_V^6 \left\{ -3s^2 f_V^2 g_K^2 \left[3s^2 + (t-u)^2 + 4M_V^2 s\right] - 4M_V^4 \left[(8g_6\,(g_6+2) - 25)\, s^2 + 3(t-u)^2\right] \right. \\ &+ 2 f_V g_K s \left[s \left\{ (26g_6 + 9)\, s^2 + (2g_6 + 7)\,(t-u)^2 \right\} - 6M_V^2 \left[(4g_6 - 3)\, s^2 + (t-u)^2\right] - 24 s M_V^4 \right] \\ &+ 2M_V^2 s \left[\left(28g_6^2 + 9\,(8g_6 - 3)\right) s^2 + \left(4g_6^2 + 13\right)(t-u)^2 \right] \\ &\left. - 4s^2 \left[3g_6\,(g_6 + 8)\, s^2 + \left(5g_6^2 + 4\right)(t-u)^2\right] - 48 M_V^6 s \right\}. \end{aligned} \tag{2.68}$$

$F^{(5)}$ is written in such a way as to make evident what controls its high-energy behaviour after the dominant $F^{(6)}$ is set to zero by taking $g_K f_V = 4g_6$. In general, these amplitudes squared grow at high energy as $(s/M_V^2)^2$, which is turned to a constant behaviour for

$$g_K f_V = 2, \quad g_6 = \frac{1}{2}. \tag{2.69}$$

In this special case the function F in eq. (2.64) acquires the form

$$F^{\text{gauge}} = 4M_V^6 \left\{ s^2 \left[s^2 - (t-u)^2\right] + 4M_V^2 s \left[2s^2 + (t-u)^2\right] - 12 M_V^4 \left[3s^2 + (t-u)^2\right] - 48 M_V^6 s \right\}. \tag{2.70}$$

2.7 *Composite* versus *gauge* models

Before studying the physical consequences for the LHC of the amplitudes calculated in the previous Sections, we consider the connection between a *composite* vector, as discussed so far, and a gauge vector of a spontaneously broken symmetry [23, 29]. For concreteness we take a gauge theory based on $G = SU(2)_L \times SU(2)_R \times SU(2)^N$ broken to the diagonal subgroup $H = SU(2)_{L+R+\ldots}$ by a generic non-linear σ-model of the form

$$\mathcal{L}_\chi = \sum_{I,J} v_{IJ}^2 \langle D_\mu \Sigma_{IJ} (D^\mu \Sigma_{IJ})^\dagger \rangle\,, \qquad \Sigma_{IJ} \to g_I \Sigma_{IJ} g_J^\dagger\,, \tag{2.71}$$

where $g_{I,J}$ are elements of the various $SU(2)$ and D_μ are covariant derivatives of G. Both the gauge couplings of the various $SU(2)$ groups and $\mathcal{L}_\chi$ are assumed to conserve parity. This *gauge* model includes as special cases or approximates via deconstruction many of the models in the literature [17, 33, 34, 35, 36, 37]. The connection between a gauge model and a composite model for the spin-1 fields is best seen at the Lagrangian level by a suitable field redefinition, as we now show.

For the clarity of exposition let us first consider the simplest $N = 1$ case, based on $SU(2)_L \times SU(2)_C \times SU(2)_R$, i.e. on the Lagrangian

$$\mathcal{L}_V^{\text{gauge}} = \mathcal{L}_\chi^{\text{gauge}} - \frac{1}{2g_C^2}\langle v_{\mu\nu}v^{\mu\nu}\rangle - \frac{1}{2g^2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle B_{\mu\nu}B^{\mu\nu}\rangle\ , \tag{2.72}$$

where

$$v_\mu = \frac{g_C}{2}v_\mu^a\tau^a \tag{2.73}$$

is the $SU(2)_C$-gauge vector and the symmetry-breaking Lagrangian is described by

$$\mathcal{L}_\chi^{\text{gauge}} = \frac{v^2}{2}\left\langle D_\mu\Sigma_{RC}\left(D^\mu\Sigma_{RC}\right)^\dagger\right\rangle + \frac{v^2}{2}\left\langle D_\mu\Sigma_{CL}\left(D^\mu\Sigma_{CL}\right)^\dagger\right\rangle. \tag{2.74}$$

Denoting collectively the three gauge vectors by

$$v_\mu^I = (W_\mu, v_\mu, B_\mu), \quad I = (L, C, R), \tag{2.75}$$

one has for the two bi-fundamental scalars Σ_{IJ}

$$D_\mu\Sigma_{IJ} = \partial_\mu\Sigma_{IJ} - iv_\mu^I\Sigma_{IJ} + i\Sigma_{IJ}v_\mu^J. \tag{2.76}$$

The Σ_{IJ} can be put in the form $\Sigma_{IJ} = \sigma_I\sigma_J^\dagger$, where σ_I are the elements of $SU(2)_I/H$, transforming under the full $SU(2)_L \times SU(2)_C \times SU(2)_R$ as $\sigma_I \to g_I\sigma_I h^\dagger$.

As the result of a gauge transformation

$$v_\mu^I \to \sigma_I^\dagger v_\mu^I\sigma_I + i\sigma_I^\dagger\partial_\mu\sigma_I \equiv \Omega_\mu^I, \quad \Sigma_{IJ} \to \sigma_I^\dagger\Sigma_{IJ}\sigma_J = 1, \tag{2.77}$$

the symmetry-breaking Lagrangian reduces to

$$\mathcal{L}_\chi^{\text{gauge}} = \frac{v^2}{2}\left\langle(\Omega_\mu^R - \Omega_\mu^C)^2\right\rangle + \frac{v^2}{2}\left\langle(\Omega_\mu^L - \Omega_\mu^C)^2\right\rangle, \tag{2.78}$$

or, after the gauge fixing $\sigma_R = \sigma_L^+ \equiv u$ and $\sigma_C = 1$, to

$$\mathcal{L}_\chi^{\text{gauge}} = v^2 \left\langle (v_\mu - i\Gamma_\mu)^2 \right\rangle + \frac{v^2}{4} \left\langle u_\mu^2 \right\rangle , \tag{2.79}$$

where

$$u_\mu = \Omega_\mu^R - \Omega_\mu^L, \quad \Gamma_\mu = \frac{1}{2i}(\Omega_\mu^R + \Omega_\mu^L) \tag{2.80}$$

coincide with the same vectors defined in Section 2.

We can finally make contact with the Lagrangian (2.9) by setting

$$v_\mu = V_\mu + i\Gamma_\mu \tag{2.81}$$

and by use of the identity [29]

$$v_{\mu\nu} = \hat{V}_{\mu\nu} - i[V_\mu, V_\nu] + \frac{i}{4}[u_\mu, u_\nu] + \frac{1}{2}(uW_{\mu\nu}u^\dagger + u^\dagger B_{\mu\nu}u). \tag{2.82}$$

With the further replacement $V_\mu \to g_C/\sqrt{2}V_\mu$, $\mathcal{L}_V^{\text{gauge}}$ coincides as anticipated with $\mathcal{L}^V$ in (2.9) for

$$g_C = \frac{1}{2g_V} \tag{2.83}$$

in the special case of (2.54) and $g_6 = 1/2, f_V = 2g_V, M_V = g_K v/2$ (or $G_V = v/2$).

2.7.1 More than a single gauge vector

To discuss the case of more than one vector, i.e. $N > 1$, one decomposes the vectors associated to $SU(2)^N$ with respect to parity as

$$\Omega_i^\mu = v_i^\mu + a_i^\mu, \quad \Omega_{P(i)}^\mu = v_i^\mu - a_i^\mu, \quad i = 1, \ldots, N, \tag{2.84}$$

so that under $SU(2)_L \times SU(2)_R$

$$v_i^\mu \to h v_i^\mu h^\dagger + ih\partial^\mu h^\dagger, \quad a_i^\mu \to h a_i^\mu h^\dagger. \tag{2.85}$$

In terms of these fields the gauge Lagrangian becomes

$$\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\text{gauge,SM}} - \sum_i \frac{1}{2g_i^2} \left[\langle (v_i^{\mu\nu} - i[a_i^\mu, a_i^\nu])^2 \rangle + \langle (D_V^\mu a_i^\nu - D_V^\nu a_i^\mu)^2 \rangle \right] , \tag{2.86}$$

where $v_i^{\mu\nu}$ are the usual field strengths and

$$D_V^\mu a_i^\nu = \partial^\mu a_i^\nu - i[v_i^\mu, a_i^\nu]. \tag{2.87}$$

At the same time, as a generalization of eq. (2.79) in the $N = 1$ case, the symmetry-breaking Lagrangian will be the sum of two separated quadratic forms in the parity-even and parity-odd fields of the type

$$\mathcal{L}_\chi^{\rm gauge} = \mathcal{L}_m^V(v_i^\mu - i\Gamma^\mu) + \mathcal{L}_m^A(u^\mu, a_i^\mu)\,. \tag{2.88}$$

The dependence of $\mathcal{L}_m^V$ on the variables $v_i^\mu - i\Gamma^\mu$ follows from (2.85).

Concentrating on the parity-even fields only, by setting

$$v_i^\mu = V_i^\mu + i\Gamma^\mu \tag{2.89}$$

and by the replacements $V_i^\mu \to g_i/\sqrt{2}V_i^\mu$, the Lagrangian of the $SU(2)_L \times SU(2)_R \times SU(2)^N$ model, restricted to the parity-even vectors, becomes a diagonal sum of $\mathcal{L}^{V_i}$, each with $g_1 = g_2 = g_4 = g_5 = 0, g_3 = -1/4, g_6 = 1/2$ and $g_{V_i} = f_{V_i}/2 = 1/g_{K_i}$, except that the V_i^μ are not mass eigenstates. Going to the mass-eigenstate basis maintains all the couplings quadratic in the V_i^μ unaltered as well as the relation $f_V = 2g_V$ for the individual mass-eingenstate vectors. On the other hand, the trilinear couplings g_{K_i} get spread among the mass eingenstates (still called V_i^μ), so that

$$\mathcal{L}_{\rm 3V} = \frac{i\hat{g}_K^{lmn}}{2\sqrt{2}} \left\langle \hat{V}_{\mu\nu}^l V_m^\mu V_n^\nu \right\rangle. \tag{2.90}$$

Picking up the lightest vector only, $i = 1$, this implies $\hat{g}_K^{111}\hat{g}_{V_1} \neq 1$, where the *hat* denotes the couplings of the physical mass eigenstates. By the orthogonality of the rotation matrix that brings to the mass basis, it is easy to prove, however, the following sum rule over the full set of vectors[3]

$$\Sigma_i \hat{g}_{V_i}\hat{g}_K^{inn} = \frac{1}{2}\Sigma_i \hat{f}_{V_i}\hat{g}_K^{inn} = 1 \tag{2.91}$$

for any fixed n. This ensures that the asymptotic behaviour of the amplitudes studied above would not be worse than in the case of a single gauge vector, but only at $s > M_{V_i}^2$ for any i.

[3] For related sum rules, see [38]

2.8 Pair production cross sections by vector boson fusion

In this Section we compute the LHC production cross section at $\sqrt{S} = 14$ TeV from VBF of two heavy vectors in the different charge configurations

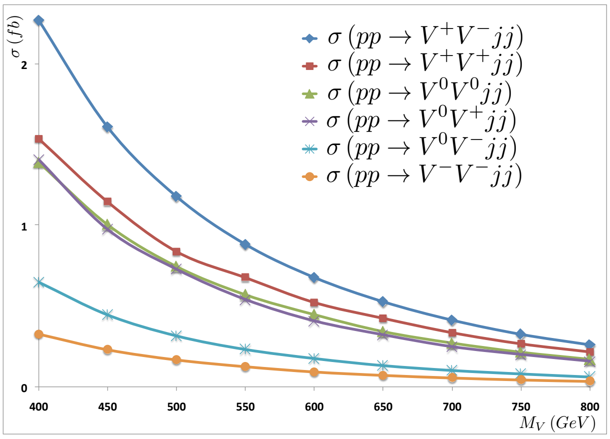
$$pp \to W^+W^-, ZZ, \gamma\gamma, \gamma Z + qq \to V^+V^- + qq \; (\to W^+Z\; W^-Z + qq), \tag{2.92}$$
$$pp \to W^+W^-, ZZ + qq \to V^0V^0 + qq \; (\to W^+W^-W^+W^- + qq), \tag{2.93}$$
$$pp \to W^\pm W^\pm + qq \to V^\pm V^\pm + qq \; (\to W^\pm Z\; W^\pm Z + qq), \tag{2.94}$$
$$pp \to W^\pm Z, W^\pm\gamma + qq \to V^\pm V^0 + qq \; (\to W^\pm Z\; W^+W^- + qq). \tag{2.95}$$

In the last step of these equations we have indicated the final state due to the largely dominant decay modes of the heavy vectors into WW or WZ (See e.g. [23]). The cross sections are summed over all the polarizations of the heavy spin-1 fields. In the calculation of the cross sections we reintroduce the hypercharge coupling $g' \neq 0$ and we make standard acceptance cuts for the forward quark jets,

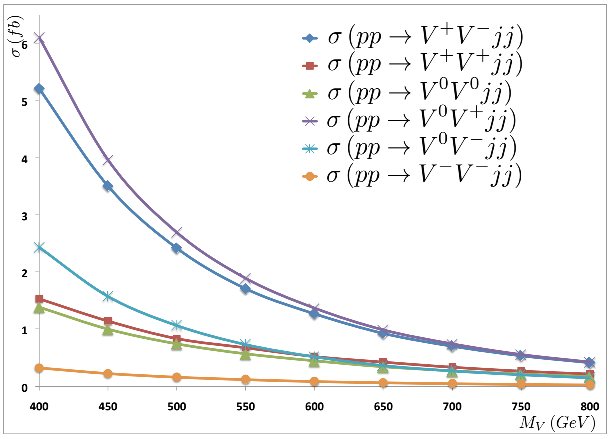
$$p_T > 30 \text{ GeV}, \quad |\eta| < 5. \tag{2.96}$$

These cross sections depend in general on a number of parameters. Fig. 1.a shows the total cross sections for the different charge channels with all the parameters fixed as in the minimal gauge model, eq. (2.54), and $G_V = g_V M_V = 200$ GeV. A value of G_V between 150 and 200 GeV keeps the elastic W_LW_L-scattering amplitude from saturating the unitarity bound below Λ, almost independently from $M_V < 1.5$ TeV [14, 23]. M_V is taken to range from 400 to 800 GeV. A value of M_V above 800 GeV would lead to a threshold for the vector-boson-fusion subprocess dangerously close to the cut-off scale of the effective Lagrangian. We have checked that the typical centre-of-mass energy of $WW \to VV$ is on average well below 2.5 TeV, even for the highest M_V that we consider.

As discussed in Sections 2.4-2.7, the parameters of the minimal gauge model damp the high energy behaviour of the different amplitudes. Not surprisingly, therefore, any deviation from them leads to significantly larger cross sections, as it may be the case already in a gauge model with more than one vector. As an example, this is shown in Fig. 1.b, where all the parameters are kept as in Fig. 1.a, except for $g_K g_V = 1/\sqrt{2}$ rather than 1, having in mind a compensation of the growing amplitudes by the occurrence of (a) significantly heavier vector(s) (See eq. 2.91). Furthermore, both in the VBF case and in the DY case, to be discussed below, it must be stressed that the



(1.a)



(1.b)

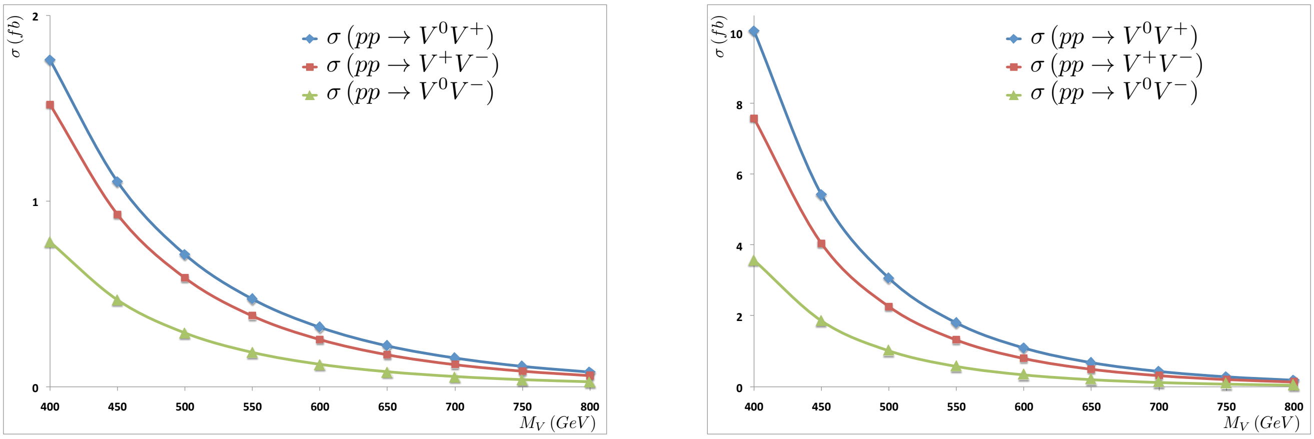
Figure 1: Total cross sections for pair production of heavy vectors via vector boson fusion in a gauge model (1.a) and a composite model (1.b) as functions of the heavy vectors masses. See text for the choice of parameters and acceptance cuts.

deviations from the minimal gauge model are quite dependent on the choice of the parameters, with cross sections that can be even higher than those in Fig. 1. In turn, these cross sections have to be considered as indicative, given the limitations of the effective Lagrangian approach.

To calculate the cross sections, we have used the matrix-element generator CalcHEP [39], which allows one to obtain the exact amplitude for a process such as $q_1q_2 \to VVq_3q_4$ via intermediate off-shell vector bosons. As a check, the results so obtained have been compared with the same cross sections in the Effective Vector Boson Approximation, using the analytic amplitudes in Sect. 2.4, for $g' = 0$ and without acceptance cuts. While being a factor of $1.5 \div 2$ systematically lower, the exact results are confirmed in their M_V-dependence and in the relative size of the different charge channels.

2.9 Drell–Yan pair production cross sections

The DY process is an additional source of V-pair production at the LHC. From the elementary parton-level amplitudes $q\bar{q} \to V^+V^-$ and $q_i\bar{q}_j \to V^{\pm}V^0$ of Section 2.6, the physical cross sections



(2.a) (2.b)

Figure 2: Total cross sections for pair production of heavy vectors via Drell–Yan $q\bar{q}$ annihilation in a gauge model (2.a) and a composite model (2.b) as functions of the heavy vectors masses. See text for the choice of parameters.

for the different charge channels

$$pp \to V^+V^-, \tag{2.97}$$

$$pp \to V^\pm V^0 \tag{2.98}$$

are readily computed. In general, the cross sections depend in this case on 3 parameters other than M_V: f_V, g_K and g_6.

As for the vector boson fusion, we show in Fig. 2.a the three cross sections for the values taken by the parameters in the minimal gauge model, $f_V g_K = 2, g_6 = 1/2$, and for $F_V = f_V M_V = 400$ GeV (corresponding to $f_V = 2g_V$ and $G_V = g_V M_V = 200$ GeV as in Fig. 1.a). On the other hand, similarly to Fig. 1.b, we show in Fig. 2.b the cross sections for $f_V g_K = \sqrt{2}, g_6 = 1/2$ and still $F_V = f_V M_V = 400$ GeV.

2.10 Same-sign di-lepton and tri-lepton events

After decay of the composite vectors,

$$V^\pm \to W^\pm Z, \quad V^0 \to W^+W^-, \tag{2.99}$$

each VV-production channel, either from VBF or from DY, leads to final states containing 2 W's and 2 Z's, from V^+V^- and $V^\pm V^\pm$, 3 W's and 1 Z, from V^+V^0, or 4 W's from V^0V^0 In fact, all

final states, except for V^+V^-, contain at least a pair of equal sign W's, i.e., after $W \to e\nu, \mu\nu$, a pair of same-sign leptons. In most cases there are at least 3 W's, i.e. also 3 leptons.

	di-leptons	tri-leptons
VBF (MGM)	16	3
DY (MGM)	5	1
VBF (comp)	28	6
DY (comp)	18	4

Table 1: Number of events with at least two same-sign leptons or three leptons (e or μ from W decays) from vector boson fusion (VBF) or Drell–Yan (DY) at LHC for $\sqrt{S} = 14$ TeV and $\int \mathcal{L}dt = 100$ fb^{-1} in the minimal gauge model (MGM) or in a composite model (comp) with the parameters as in Figs. 1-2 and $M_V = 500$ GeV.

	di-leptons(%)	tri-leptons(%)
V^0V^0	8.9	3.2
$V^\pm V^\pm$	4.5	-
$V^\pm V^0$	4.5	1.0

Table 2: Cumulative branching ratios for at least two same-sign leptons or three leptons (e or μ) in the W-decays from two vectors in the given charge configuration.

At the LHC with an integrated luminosity of 100 inverse femtobarns and $\sqrt{S} = 14$ TeV, putting together all the different charge configurations, one obtains from $W \to e\nu, \mu\nu$ decays the number of same-sign di-leptons and tri-lepton events given in Table 1 for $M_V = 500$ GeV. The other parameters are fixed as in the Minimal Gauge Model (and labelled MGM) or as in Figs. 1.b-2.b for VBF and for DY in the previous two Sections (and labelled *comp*). These numbers of events are based on the cross sections in Figs. 1-2 and on the branching ratios for the various charge channels listed in Table 2. The numbers of events for different values of M_V are also easily obtained. As already noticed, depending on the parameters, the number of events in the *composite* case could also be significantly higher. No attempt is made, at this stage, to compare the signal with the background from SM sources. To see if a signal can be observed a careful analysis will be required, with a high cut on the scalar sum, H_t, of all the transverse momenta and of the missing energy in each event probably playing a crucial role. The use of the leptonic decays of the Z might also be important.

3 A "composite" scalar-vector system at the LHC

In this chapter we are interested to the case in which both a vector and a light scalar are relevant with a mass below the cutoff $\Lambda \approx 3$ TeV. In this case the role of unitarization of the different scattering channels is played both by the scalar and the vector (an example of this phenomenon is discussed for Technicolor models in [40]). In particular, the unitarity in the elastic longitudinal gauge boson scattering does not completely constrain the couplings of the scalar and the vector to the gauge bosons, but implies a relation among them. Therefore in this case there is a wider region in the parameter space that is reasonable from the point of view of unitarity, at least in the elastic channel. In this framework we are interested to study the phenomenology of the associated scalar-vector production, that is peculiar to the present case[4].

3.1 The basic Lagrangian

We are interested to study a scalar-vector system in the framework of Strongly Interacting EWSB by adopting an approach as model independent as possible. Nevertheless, for our approach to make sense, we have to make some assumptions. One way to state these assumptions is the following:

1. Before weak gauging, the Lagrangian responsible for EWSB has a $SU(2)_L \times SU(2)^N \times SU(2)_R$ global symmetry, with $SU(2)^N$ gauged, spontaneously broken to the diagonal $SU(2)_d$ by a generic non-linear sigma model.

2. Only one vector triplet V_μ^a of the $SU(2)^N$ gauge group has a mass below the cutoff $\Lambda \approx 3\,\text{TeV}$, while all the other heavy vectors can be integrated out. Furthermore the new vector triplet V_μ^a couples to fermions only through the mixing with the weak gauge bosons of $SU(2)_L \times U(1)_Y$, $Y = T_{3R} + 1/2(B-L)$.

3. The spectrum also contains a scalar singlet of $SU(2)_d$ with a relatively low mass $m_h \lesssim v$.

We believe that these assumptions may represent a physically interesting situation. Under these assumptions, it follows that the interactions among the composite singlet scalar, composite

[4]We shall not impose the constraints coming from the EWPT since further effects can be present, e.g. due to new fermionic degrees of freedom, that obscure their interpretation and/or a strong sensitivity to the physics at the cutoff may be involved which we do not pretend to control.

triplet heavy vectors, Goldstone bosons and the SM gauge fields can be described by a model independent $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ Chiral Lagrangian given by:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\chi + \mathcal{L}^V + \mathcal{L}_h + \mathcal{L}_{h-V}, \tag{3.1}$$

where the different terms will be explained in the following.

The term $\mathcal{L}_\chi$ is the usual lowest order chiral Lagrangian for the $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ Goldstone fields with the addition of the invariant kinetic terms for the W and B bosons and has the following form:

$$\mathcal{L}_\chi = \frac{v^2}{4}\left\langle D_\mu U \left(D^\mu U\right)^\dagger\right\rangle - \frac{1}{2g^2}\left\langle W_{\mu\nu}W^{\mu\nu}\right\rangle - \frac{1}{2g'^2}\left\langle B_{\mu\nu}B^{\mu\nu}\right\rangle . \tag{3.2}$$

The Lagrangian $\mathcal{L}^V$ which contains the kinetic and mass terms for the heavy spin-1 fields, the vector self-interactions as well as the interactions of these vectors with the Goldstone bosons and SM gauge fields is given by:

$$\begin{aligned}\mathcal{L}^V \;=\; & -\frac{1}{4}\left\langle \hat{V}^{\mu\nu}\hat{V}_{\mu\nu}\right\rangle + \frac{M_V^2}{2}\left\langle V^\mu V_\mu\right\rangle - \frac{ig_V}{2\sqrt{2}}\left\langle \hat{V}_{\mu\nu}[u^\mu, u^\nu]\right\rangle - \frac{g_V}{\sqrt{2}}\left\langle \hat{V}_{\mu\nu}\left(uW^{\mu\nu}u^\dagger + u^\dagger B^{\mu\nu}u\right)\right\rangle \\ & + \frac{i}{2}\left\langle V_\mu V_\nu\left(uW^{\mu\nu}u^\dagger + u^\dagger B^{\mu\nu}u\right)\right\rangle + \frac{ig_K}{4\sqrt{2}}\left\langle \hat{V}_{\mu\nu}[V^\mu, V^\nu]\right\rangle - \frac{1}{8}\left\langle [V_\mu, V_\nu][u^\mu, u^\nu]\right\rangle \\ & + \frac{g_V^2}{8}\left\langle [u_\mu, u_\nu][u^\mu, u^\nu]\right\rangle .\end{aligned} \tag{3.3}$$

The Lagrangian $\mathcal{L}_h$ includes the kinetic and mass terms for the scalar as well as the interactions of this scalar with the Goldstone bosons and SM gauge fields and is given by:

$$\mathcal{L}_h = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{m_h^2}{2}h^2 + \frac{v^2}{4}\left\langle D_\mu U \left(D^\mu U\right)^\dagger\right\rangle \left(2a\frac{h}{v} + b\frac{h^2}{v^2}\right) . \tag{3.4}$$

The term $\mathcal{L}_{h-V}$ is the scalar-vector interaction Lagrangian:

$$\mathcal{L}_{h-V} = \frac{dv}{8g_V^2} h \left\langle V_\mu V^\mu\right\rangle . \tag{3.5}$$

The light scalar that we are considering could be a Strongly Interacting Light Higgs (SILH) boson in the sense of [10] or a more complicated object arising from an unknown strong dynamics. The

couplings of this particle to the SM particles and to the heavy vector V will be strongly related to the mechanism that generates it. The measurement of the different cross sections that are sensitive to the different couplings, hopefully at the LHC but eventually also at a future Linear Collider, could give information about this mechanism.

We show in Appendix that the Lagrangian (3.1), for the special values

$$a = \frac{1}{2}\,, \quad b = \frac{1}{4}\,, \quad d = 1\,, \quad g_K = \frac{1}{g_V}\,, \quad g_V = \frac{v}{2M_V}\,, \tag{3.6}$$

is obtained from a gauge theory based on $SU(2)_L \times SU(2)_C \times U(1)_Y$ spontaneously broken by two Higgs doublets (with the same VEV) in the limit $m_H \gg \Lambda$ for the mass of the L-R-parity odd scalar H[5].

3.2 Two body $W_L W_L$ scattering amplitudes

In this Section we compute the scattering amplitudes:

$$\begin{array}{lcl} \mathcal{A}\left(W_L^a W_L^b \to W_L^c W_L^d\right) & & \mathcal{A}\left(\pi^a\pi^b \to \pi^c\pi^d\right) \\ \mathcal{A}\left(W_L^a W_L^b \to V_L^c V_L^d\right) & \Longrightarrow & -\mathcal{A}\left(\pi^a\pi^b \to V_L^c V_L^d\right) \\ \mathcal{A}\left(W_L^a W_L^b \to hh\right) & \sqrt{s} \gg M_W & -\mathcal{A}\left(\pi^a\pi^b \to hh\right) \\ \mathcal{A}\left(W_L^a W_L^b \to V_L^c h\right) & & -\mathcal{A}\left(\pi^a\pi^b \to V_L^c h\right)\,, \end{array} \tag{3.7}$$

where we make use of the Equivalence Theorem to relate the scattering amplitudes involving the Goldstone bosons with the high energy limit of those ones involving the longitudinal polarization of the weak gauge bosons[6]. As before, to simplify the explicit formulae we take the limit $g' = 0$ (that implies $Z \approx W^3$) so that the $SU(2)_{L+R}$ invariance is preserved by the scattering amplitudes.

We can study the four processes one by one.

- $\pi^a\pi^b \to \pi^c\pi^d$ scattering amplitude

  Using the $SU(2)_{L+R}$ invariance and the Bose symmetry the amplitude for the four pion

[5]As we discuss in Appendix the mass of the L-R-parity odd scalar H can be simply raised above the cut-off without any further hypothesis on the low energy physics.

[6]The minus sign in the last three amplitudes in (3.7) is due to the fact that the Equivalence Theorem has a factor $(-i)^N$ where N is the number of external longitudinal vector bosons.

scattering can be written in the form

$$\mathcal{A}\left(\pi^a\pi^b \to \pi^c\pi^d\right) = \mathcal{A}\left(s,t,u\right)^{\pi\pi\to\pi\pi}\,\delta^{ab}\delta^{cd} + \mathcal{A}\left(t,s,u\right)^{\pi\pi\to\pi\pi}\,\delta^{ab}\delta^{cd} + \mathcal{A}\left(u,t,s\right)^{\pi\pi\to\pi\pi}\,\delta^{ab}\delta^{cd}\,. \tag{3.8}$$

It receives contributions from the four pion contact interaction π^4 and from the exchange of W, V and h. The contribution coming from the exchange of a W boson is sub-leading in the sense of the Equivalence Theorem, i.e. is of order $M_W/\sqrt{s}$ and therefore we can write

$$\mathcal{A}\left(\pi^a\pi^b \to \pi^c\pi^d\right) = \mathcal{A}\left(\pi^a\pi^b \to \pi^c\pi^d\right)_{\pi^4} + \mathcal{A}\left(\pi^a\pi^b \to \pi^c\pi^d\right)_{V} + \mathcal{A}\left(\pi^a\pi^b \to \pi^c\pi^d\right)_{h}\,, \tag{3.9}$$

so that we obtain

$$\mathcal{A}\left(s,t,u\right)^{\pi\pi\to\pi\pi} = \frac{s}{v^2} + \frac{g_V^2 M_V^2}{v^4}\Big[-3s + M_V^2\left(\frac{(u-s)}{t-M_V^2} + \frac{(t-s)}{u-M_V^2}\right)\Big] - \frac{a^2}{v^2}\left(\frac{s^2}{s-m_h^2}\right)\,. \tag{3.10}$$

- $\pi^a\pi^b \to V_L^c V_L^d$ scattering amplitude
  The amplitude can be reduced to

$$\mathcal{A}\left(\pi^a\pi^b \to V_L^c V_L^d\right) = \mathcal{A}\left(s,t,u\right)^{\pi\pi\to VV}\,\delta^{ab}\delta^{cd} + \mathcal{B}\left(s,t,u\right)^{\pi\pi\to VV}\,\delta^{ab}\delta^{cd} + \mathcal{B}\left(s,u,t\right)^{\pi\pi\to VV}\,\delta^{ab}\delta^{cd}\,. \tag{3.11}$$

It receives contributions from the π^2V^2 contact interaction and the exchange of π, V and h

$$\begin{aligned}\mathcal{A}\left(\pi^a\pi^b \to VV\right) &= \mathcal{A}\left(\pi^a\pi^b \to VV\right)_{\pi^2V^2} + \mathcal{A}\left(\pi^a\pi^b \to VV\right)_{\pi} \\ &\quad + \mathcal{A}\left(\pi^a\pi^b \to VV\right)_{V} + \mathcal{A}\left(\pi^a\pi^b \to VV\right)_{h}\,.\end{aligned} \tag{3.12}$$

The explicit forms obtained for $\mathcal{A}\left(s,t,u\right)^{\pi\pi\to VV}$ and $\mathcal{B}\left(s,t,u\right)^{\pi\pi\to VV}$ are

$$\mathcal{A}\left(s,t,u\right)^{\pi\pi\to VV} = \frac{g_V^2 M_V^2 s}{v^4\left(s-4M_V^2\right)}\Big[\frac{\left(t+M_V^2\right)^2}{t} + \frac{\left(u+M_V^2\right)^2}{u}\Big] + \frac{ad}{2v^2}\left(\frac{s}{s-m_h^2}\right)\left(s-2M_V^2\right)\,, \tag{3.13}$$

$$\mathcal{B}\left(s,t,u\right)^{\pi\pi\to VV} = \frac{t-u}{2v^2} - \frac{g_V^2 M_V^2 s\left(u+M_V^2\right)^2}{v^4 u\left(s-4M_V^2\right)} + \frac{s\left(u-t\right)}{4v^2M_V^2}\left(g_V g_K\frac{s+2M_V^2}{s-M_V^2} - 1\right)\,. \tag{3.14}$$

- $\pi^a\pi^b \to hh$ scattering amplitude
  The amplitude can be written as

$$\mathcal{A}\left(\pi^a\pi^b \to hh\right) = \mathcal{A}\left(s,t,u\right)^{\pi\pi\to hh}\,\delta^{ab}\,. \tag{3.15}$$

This amplitude receives contributions from the π^2h^2 contact interaction and the exchange of π and h

$$\mathcal{A}\left(\pi^a\pi^b \to hh\right) = \mathcal{A}\left(\pi^a\pi^b \to hh\right)_{\pi^2h^2} + \mathcal{A}\left(\pi^a\pi^b \to hh\right)_{\pi} + \mathcal{A}\left(\pi^a\pi^b \to hh\right)_{h}\,. \tag{3.16}$$

In this case $\mathcal{A}(s,t,u)^{\pi\pi\to hh}$ is given by

$$\mathcal{A}(s,t,u)^{\pi\pi\to hh} = -\frac{1}{v^2}\left(s\left(b-a^2\right)+\frac{3asm_h^2}{2\left(s-m_h^2\right)}-2a^2m_h^2+\frac{a^2m_h^4}{t}+\frac{a^2m_h^4}{u}\right). \tag{3.17}$$

- $\pi^a\pi^b \to V_L^c h$ scattering amplitude

  The $SU(2)_{L+R}$ invariance implies

$$\mathcal{A}\left(\pi^a\pi^b \to V_L^c h\right) = \mathcal{A}(s,t,u)^{\pi\pi\to Vh}\,\epsilon^{abc}. \tag{3.18}$$

The amplitude receives contributions from the exchange of π and V

$$\mathcal{A}\left(\pi^a\pi^b \to V_L^c h\right) = \mathcal{A}\left(\pi^a\pi^b \to V_L^c h\right)_\pi + \mathcal{A}\left(\pi^a\pi^b \to V_L^c h\right)_V \tag{3.19}$$

so that the explicit value of $\mathcal{A}(s,t,u)^{\pi\pi\to Vh}$ is

$$\begin{aligned}\mathcal{A}(s,t,u)^{\pi\pi\to Vh} &= \frac{i\,(t-u)}{2v\sqrt{\left(M_V^2+m_h^2-s\right)^2-4m_h^2M_V^2}}\left[\frac{d}{4g_VM_V}\frac{s}{s-M_V^2}\left(m_h^2-M_V^2-s\right)\right.\\ &\left.+\frac{2ag_VM_V}{v^2tu}\left[m_h^2M_V^2\left(m_h^2-M_V^2+s\right)+tu\left(M_V^2-m_h^2+s\right)\right]\right].\end{aligned} \tag{3.20}$$

3.3 Asymptotic amplitudes and parameter constraints

In the very high energy limit in which $s \gg M_V^2 \gg m_h^2$ we can summarize the amplitudes (3.10), (3.13), (3.14), (3.17) and (3.20) as follows:

$$\mathcal{A}(s,t,u)^{\pi\pi\to\pi\pi} \approx \frac{s}{v^2}\left(1-a^2-\frac{3g_V^2M_V^2}{v^2}\right)+\frac{g_V^2M_V^4}{v^4}\left[\left(\frac{(u-s)}{t}+\frac{(t-s)}{u}\right)\right], \tag{3.21a}$$

$$\mathcal{A}(s,t,u)^{\pi\pi\to VV} \approx \left(\frac{ad}{2v^2}-\frac{1}{4v^2}\right)\left(s-2M_V^2\right), \tag{3.21b}$$

$$\begin{aligned}\mathcal{B}(s,t,u)^{\pi\pi\to VV} &\approx \frac{u-t}{2v^2}\left[\frac{s}{2M_V^2}\left(g_Vg_K-1\right)-1+\frac{3g_Vg_K}{2}\left(1+\frac{M_V^2}{s}\right)\right]\\ &-\frac{g_V^2M_V^2u}{v^4}\left(1+\frac{4M_V^2}{s}+\frac{2M_V^2}{u}\right),\end{aligned} \tag{3.21c}$$

$$\mathcal{A}(s,t,u)^{\pi\pi\to hh} \approx -\frac{1}{v^2}\left[\left(b-a^2\right)s+\frac{am_h^2}{2}(3-4a)\right], \tag{3.21d}$$

$$\begin{aligned}\mathcal{A}(s,t,u)^{\pi\pi\to Vh} &\approx \frac{ig_VM_V\,(t-u)}{v}\left[\frac{a}{v^2}-\frac{d}{8g_V^2M_V^2}\right]\\ &+\frac{ig_VM_V\,(t-u)}{vs}\left[\frac{a}{v^2}\left(M_V^2-m_h^2\right)+\frac{d}{8g_V^2M_V^2}\left(m_h^2-2M_V^2\right)\right].\end{aligned} \tag{3.21e}$$

For generic values of the parameters, all these amplitudes grow with the c.o.m. energy like s except $\mathcal{B}(s,t,u)^{\pi\pi\to VV}$ that grows like s^2. On the other hand, with the parameters as in (3.6) the amplitudes reduce to

$$\mathcal{A}(s,t,u)^{\pi\pi\to\pi\pi} \approx \frac{M_V^2}{4v^2}\left[\left(\frac{(u-s)}{t}+\frac{(t-s)}{u}\right)\right]+O\left(\frac{m_h^2}{v^2}\right), \tag{3.22a}$$

$$\mathcal{A}(s,t,u)^{\pi\pi\to VV} \approx O\left(\frac{m_h^2}{v^2}\right), \tag{3.22b}$$

$$\mathcal{B}(s,t,u)^{\pi\pi\to VV} \approx -\frac{t}{4v^2}-\frac{M_V^2}{4v^2}\left(\frac{u+3t}{s}+2\right), \tag{3.22c}$$

$$\mathcal{A}(s,t,u)^{\pi\pi\to hh} \approx -\frac{m_h^2}{4v^2}, \tag{3.22d}$$

$$\mathcal{A}(s,t,u)^{\pi\pi\to Vh} \approx \frac{iM_V^2\,(u-t)}{4v^2 s}+O\left(\frac{m_h^2}{v^2}\right). \tag{3.22e}$$

From the last relations we see that with the choice (3.6) of the parameters, that corresponds to the choice of the $SU(2)_L \times SU(2)_C \times U(1)_Y$ gauge model spontaneously broken by two Higgs doublets in the limit of very heavy L-R-parity odd scalar H, all the amplitudes except for $\mathcal{B}(s,t,u)^{\pi\pi\to VV}$ have a constant asymptotic behavior. As shown in the Appendix 4 if we also add to the spectrum the H scalar we can also regulate the $\mathcal{B}(s,t,u)^{\pi\pi\to VV}$ amplitude making the theory asymptotically well behaved and fully perturbative.

The choice of parameters as in (3.6) is however too restrictive. Other than $g_V g_K = 1$, so that the $\pi\pi \to VV$ scattering amplitude grows at most like s, we only pretend that the exchange of the scalar and of the vector lead together to a good asymptotic behavior of elastic $W_L W_L$ scattering, i.e.

$$a=\sqrt{1-\frac{3G_V^2}{v^2}}, \qquad G_V \equiv g_V M_V. \tag{3.23}$$

The processes (3.7) are all important at the LHC in order to understand the underlying mechanism that can generate the spectrum that we consider. In fact the pair production of new states can be very useful to measure the different couplings and to constrain the parameter space. Both the scalar and vector pair productions have been recently studied in [32] and [20], respectively. The phenomenology studied in those works changes as follows in the present approach:

- Scalar pair production
  Equations (3.16) shows that there are no contributions of the heavy vector to the scalar pair production so that the results of [32] exactly hold also in this case.

- Vector pair production
  From equation (3.12) we see that there is a contribution to the heavy vectors pair production coming from the scalar exchange. However, having imposed in this Chapter relation (3.23) so that $\mathcal{A}(W_L W_L \to W_L W_L) \simeq const$ at high energy, one has $G_V \leq v/\sqrt{3}$, which leads to a $W_L W_L \to VV$ cross section well below the values found in Chapter 2.

It remains to study the associated Vh production not considered before. The associated production can be generated both by Vector Boson Fusion (VBF) and by Drell-Yan (DY) $q\bar{q}$ annihilation. In the next section we discuss the total cross sections for the associated production by VBF and by DY.

3.4 Associated production of Vh total cross sections

In this section we discuss the total cross section for the associated Vh production of the heavy vector and the light scalar. There are three possible final states for the associated production, corresponding to the three charge states of the V: hV^-, hV^0 and hV^+. According to the constraints discussed in the previous Section on the parameter space we can compute the total cross sections for some reference values of the independent parameters that we choose to be G_V and d. Some values of the total cross sections at the LHC for $\sqrt{s} = 14$ TeV for different values of the parameters and for a scalar mass $m_h = 180$ GeV are listed in Tables 3, 4 and 5 for the production of hV^-, hV^0 and hV^+ respectively. We have chosen $m_h = 180$ GeV to maximize both the total cross sections and the branching ratio for $h \to W^+W^-$. In this case signals of the associated productions can appear in the multi-lepton channels. In particular if the final state contains at least a pair of equal sign W's there can be signals in the same-sign di-lepton and tri-lepton final states from W decays that are much simpler to be separated from the background than those corresponding to the hadronic final states. Obviously different values of m_h are possible: in that case the detection of a signal can be disfavored by the large branching ratio for $h \to b\bar{b}$ for $m_h < 2M_W$, by the large branching ratio for $h \to ZZ$ for $m_h > 2M_Z$ and by the small cross sections for $m_h \gtrsim 250$ GeV (see Fig. 4).

The total cross sections have been computed using the Matrix Element Generator CalcHEP [39] with the CTEQ5M NLO parton distribution functions, the model was implemented in it using the FeynRules Mathematica package [41]. For the calculation of the VBF total cross sections the

G_V	a	d	VBF (fb)	DY (fb)
$\sqrt{5}v/4$	1/4	0	0.05	0
$\sqrt{5}v/4$	1/4	1	0.09	3.31
$\sqrt{5}v/4$	1/4	2	0.62	13.24
$v/2$	1/2	0	0.15	0
$v/2$	1/2	1	0.05	4.14
$v/2$	1/2	2	0.56	16.56
$v/\sqrt{6}$	$1/\sqrt{2}$	0	0.20	0
$v/\sqrt{6}$	$1/\sqrt{2}$	1	0.08	6.20
$v/\sqrt{6}$	$1/\sqrt{2}$	2	0.89	24.80

(3.a)

G_V	a	d	VBF (fb)	DY (fb)
$\sqrt{5}v/4$	1/4	0	0.02	0
$\sqrt{5}v/4$	1/4	1	0.08	1.23
$\sqrt{5}v/4$	1/4	2	0.49	4.92
$v/2$	1/2	0	0.07	0
$v/2$	1/2	1	0.06	1.54
$v/2$	1/2	2	0.48	6.16
$v/\sqrt{6}$	$1/\sqrt{2}$	0	0.09	0
$v/\sqrt{6}$	$1/\sqrt{2}$	1	0.09	2.30
$v/\sqrt{6}$	$1/\sqrt{2}$	2	0.75	9.20

(3.b)

Table 3: Total cross sections for the associated production of hV^- final state by VBF and DY at the LHC for $\sqrt{s} = 14$ TeV as functions of the different parameters for $M_V = 700$ GeV (3.a) and $M_V = 1$ TeV (3.b). The parameter a is fixed by the value of G_V (and vice versa) according to equation (3.23).

G_V	a	d	VBF(fb)	DY(fb)
$\sqrt{5}v/4$	1/4	0	0.08	0
$\sqrt{5}v/4$	1/4	1	0.14	6.14
$\sqrt{5}v/4$	1/4	2	0.99	24.56
$v/2$	1/2	0	0.24	0
$v/2$	1/2	1	0.08	7.67
$v/2$	1/2	2	0.90	30.68
$v/\sqrt{6}$	$1/\sqrt{2}$	0	0.32	0
$v/\sqrt{6}$	$1/\sqrt{2}$	1	0.13	11.51
$v/\sqrt{6}$	$1/\sqrt{2}$	2	1.42	46.04

(4.a)

G_V	a	d	VBF(fb)	DY(fb)
$\sqrt{5}v/4$	1/4	0	0.04	0
$\sqrt{5}v/4$	1/4	1	0.13	2.43
$\sqrt{5}v/4$	1/4	2	0.79	9.74
$v/2$	1/2	0	0.11	0
$v/2$	1/2	1	0.09	3.04
$v/2$	1/2	2	0.78	12.16
$v/\sqrt{6}$	$1/\sqrt{2}$	0	0.15	0
$v/\sqrt{6}$	$1/\sqrt{2}$	1	0.15	4.57
$v/\sqrt{6}$	$1/\sqrt{2}$	2	1.22	18.28

(4.b)

Table 4: Total cross sections for the associated production of hV^0 final state by VBF and DY at the LHC for $\sqrt{s} = 14$ TeV as functions of the different constants for $M_V = 700$ GeV (4.a) and $M_V = 1$ TeV (4.b). The parameter a is fixed by the value of G_V (and vice versa) according to equation (3.23).

acceptance cuts $p_{Tj} > 30$ GeV and $|\eta| < 5$ for the forward quark jets have been imposed. From the tables we immediately see that the DY total cross sections are much greater than the corresponding VBF ones. This is due in part to the structure of the phase space, which for the DY is a $2 \to 2$ and for the VBF is a $2 \to 4$ and in part to the structure of the squared amplitude which for the

G_V	a	d	VBF(fb)	DY(fb)
$\sqrt{5}v/4$	$1/4$	0	0.10	0
$\sqrt{5}v/4$	$1/4$	1	0.18	7.30
$\sqrt{5}v/4$	$1/4$	2	1.28	29.20
$v/2$	$1/2$	0	0.33	0
$v/2$	$1/2$	1	0.10	9.12
$v/2$	$1/2$	2	1.15	36.48
$v/\sqrt{6}$	$1/\sqrt{2}$	0	0.43	0
$v/\sqrt{6}$	$1/\sqrt{2}$	1	0.17	13.68
$v/\sqrt{6}$	$1/\sqrt{2}$	2	1.82	54.72

(5.a)

G_V	a	d	VBF(fb)	DY(fb)
$\sqrt{5}v/4$	$1/4$	0	0.05	0
$\sqrt{5}v/4$	$1/4$	1	0.18	3.03
$\sqrt{5}v/4$	$1/4$	2	1.10	12.12
$v/2$	$1/2$	0	0.16	0
$v/2$	$1/2$	1	0.12	3.79
$v/2$	$1/2$	2	1.07	15.16
$v/\sqrt{6}$	$1/\sqrt{2}$	0	0.22	0
$v/\sqrt{6}$	$1/\sqrt{2}$	1	0.20	5.69
$v/\sqrt{6}$	$1/\sqrt{2}$	2	1.66	22.76

(5.b)

Table 5: Total cross sections for the associated production of hV^+ final state by VBF and DY at the LHC for $\sqrt{s} = 14$ TeV as functions of the different constants for $M_V = 700$ GeV (5.a) and $M_V = 1$ TeV (5.b). The parameter a is fixed by the value of G_V (and vice versa) according to equation (3.23).

DY is proportional to

$$|\mathcal{A}\left(q\bar{q} \to Vh\right)|^2 \propto g_V^2 \frac{d^2}{g_V^4} = \frac{d^2}{g_V^2}\,, \tag{3.24}$$

while the VBF squared amplitude includes a strong dependance on $d - a$ and has a more complicated structure than the DY squared amplitude.

The Figure 3 shows the total cross sections for the DY associated production at the LHC for $\sqrt{s} = 14$ TeV as functions of the heavy vector mass for different values of the parameter G_V (and therefore of a according to (3.23)). We see that even for $d = 1$ that corresponds to the choice of the gauge model coupling (see App. 4), the total cross sections are of order of 10 fb for a vector mass between 500 GeV and 800 GeV. Furthermore, since the DY total cross sections grow with d^2, deviations from $d = 1$ could result in a strong increase of the values given in Figure 3.

Finally, to give an idea of the dependence of the total cross sections on the scalar mass m_h, we plot in Fig. 4 the total cross sections for the Vh associated production as functions of the scalar mass for $150GeV < m_h < 300GeV$. From Fig. 4 we immediately see that the total cross sections have almost halved, going from $m_h = 180$ GeV to $m_h = 270$ GeV. Taking also into account the relevant branching ratio of h we can conclude that a scalar with a mass between $2M_W$ and $2M_Z$ is the most favorable situation to find a signal of the associated production, while it can be much

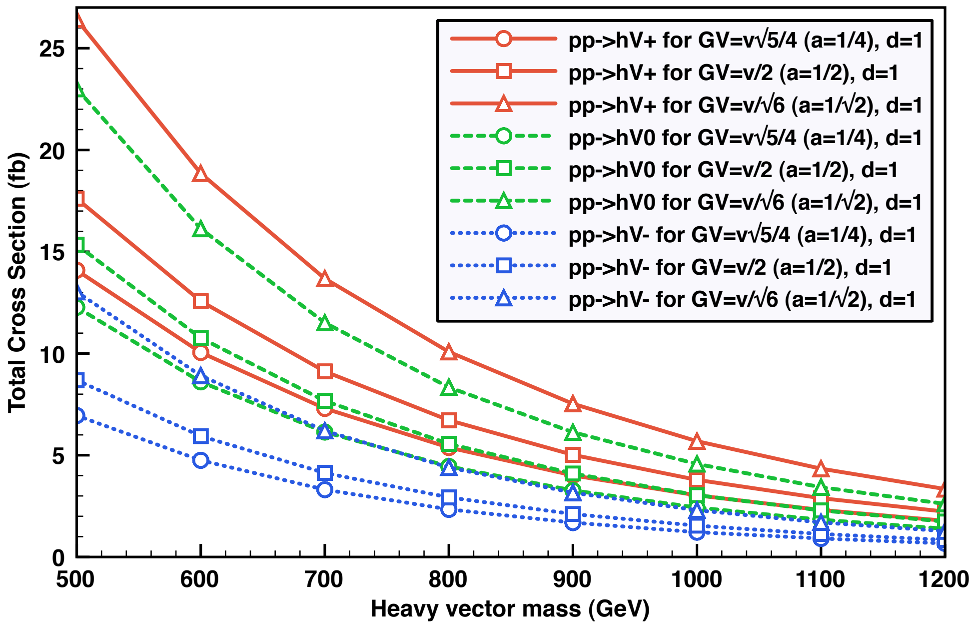


Figure 3: Total cross sections for the Vh associated productions via Drell–Yan $q\bar{q}$ annihilation as functions of the heavy vector mass at the LHC for $\sqrt{s} = 14$ TeV, for $m_h = 180$ GeV, for different values of G_V (corresponding to different values of a according to (3.23)) and for $d = 1$. Since the DY total cross sections are proportional to d^2 the results can be simply generalized to different values of d.

more difficult to access a signal for $m_h < 2M_W$ or $m_h > 2M_Z$ than for $2M_W < m_h < 2M_Z$.

3.5 Same-sign di-lepton and tri-lepton events

The number of multi-lepton events is strongly dependent on the decay modes of the light scalar and the heavy vector. As the vector couples to the fermions only via the mixing with the weak gauge bosons, the decay width of V into fermions is strongly suppressed with respect to the decay width into gauge bosons. In the limit $g' \approx 0$ we can write [18]

$$\frac{\Gamma\left(V^0 \to \bar{\psi}\psi\right)}{\Gamma\left(V^0 \to W_L^+ W_L^-\right)} \approx \frac{4M_W^4}{M_V^4}\,, \tag{3.25}$$

so that we can take the branching ratios

$$\mathrm{BR}\left(V^+ \to W_L^+ Z_L\right) \approx \mathrm{BR}\left(V^0 \to W_L^+ W_L^-\right) \approx 1\,. \tag{3.26}$$

For what concerns the scalar h we neglect $\Gamma\left(h \to \bar{\psi}\psi\right)$ with respect to $\Gamma\left(h \to W^+ W^-\right)$.

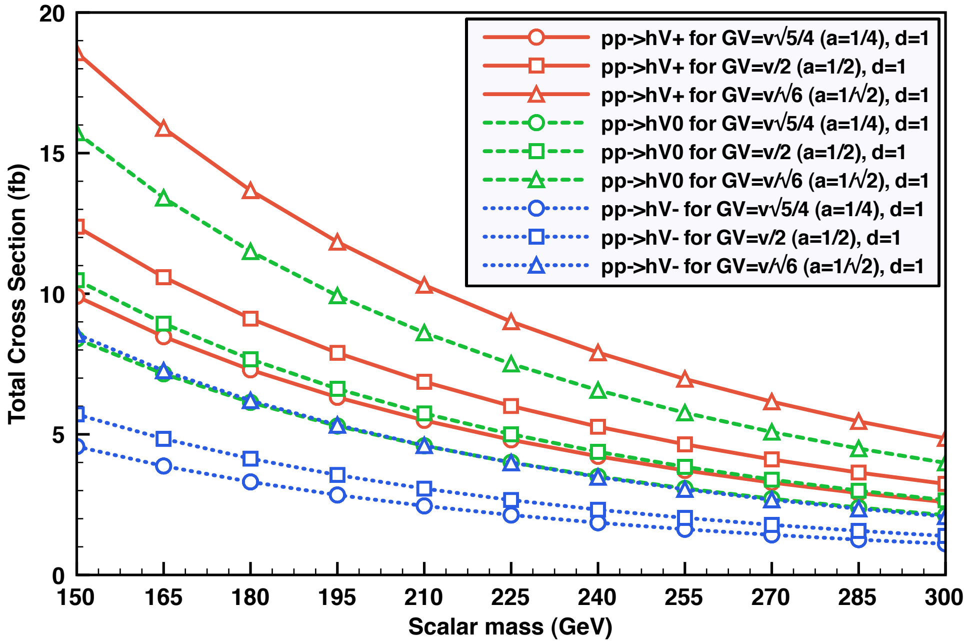


Figure 4: Total cross sections for the Vh associated productions via Drell–Yan $q\bar{q}$ annihilation as functions of the scalar mass at the LHC for $\sqrt{s} = 14$ TeV, for $M_V = 700$ GeV, for different values of G_V (corresponding to different values of a according to (3.23)) and for $d = 1$. Since the DY total cross sections are proportional to d^2 the results can be simply generalized to different values of d.

Decay Mode	di-leptons (%)	tri-leptons (%)
$V^0 h \to W^+W^-W^+W^-$	8.9	3.2
$V^\pm h \to W^\pm Z W^+W^-$	4.5	1.0

Table 6: Decay modes and cumulative branching ratios for the different charge configurations of the hV system assuming $BR(h \to W^+W^-) \approx 1$. For the same sign di-lepton and tri-lepton branching ratios we consider only the e and μ leptons coming from the W decays.

Using the values of the branching fractions given in Table 6 and a reference integrated luminosity of $\int \mathcal{L}dt = 100$ fb^{-1} we obtain the total number of same sign di-lepton and tri-lepton events given in Table 7.

G_V	a	di-leptons	tri-leptons
$\sqrt{5}v/4$	$1/4$	102.4	30.3
$v/2$	$1/2$	128.0	37.8
$v/\sqrt{6}$	$1/\sqrt{2}$	192.0	56.7

(7.a)

G_V	a	di-leptons	tri-leptons
$\sqrt{5}v/4$	$1/4$	41.0	12.0
$v/2$	$1/2$	51.0	15.1
$v/\sqrt{6}$	$1/\sqrt{2}$	76.6	22.6

(7.b)

Table 7: Total number of same sign di-lepton and tri-lepton events (e or μ from W decays) for the DY Vh associated production at the LHC for $\sqrt{s} = 14$ TeV and $\int \mathcal{L}dt = 100$ fb^{-1} for $M_V = 700$ GeV (7.a) and $M_V = 1$ TeV (7.b) for different values of the parameter G_V (or a according to equation (3.23)) and for $d = 1$. Since the DY total cross sections are proportional to d^2 the results can simply be generalized to different values of d.

4 Conclusions

While being disfavored relative to a weak coupling picture, the possibility that EWSB be due to a new strong interaction at about $4\pi v$ remains open. As a matter of fact the difficulties that different models of this kind encounter in reproducing the experimental data may have something to do with the lack of reliable computational tools in strong coupling theories. In turn this may obscure the emergence of the right dynamics or even of the right explicit model for EWSB. The lack so far of a thorough experimental exploration of the energy range at or well above the Fermi scale should also not be forgotten. A way to provisionally overcome this difficult situation may be offered by the EWCL with the inclusion of some "composite" particles. EWCL are a minimal way to describe massive vectors consistently with gauge invariance. Their problems and their limitations are well known. Yet they offer a conceptual framework to describe the phenomenology of a strong dynamics maybe responsible of EWSB in a way that may help unravelling its structure.

In the framework of a strongly interacting dynamics for EWSB, composite heavy vector and scalar states may exist. The interactions among themselves and with the Standard Model gauge bosons can be described by a $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ Effective Chiral Lagrangian. These composite heavy vector and composite scalar resonances play a special role in preserving unitarity in longitudinal WW scattering. In the first part of the thesis we have consider the case in which a $SU(2)_{L+R}$-triplet of composite vectors with a mass lower than $\Lambda \approx 4\pi v$ is relevant. The pair production of such composite vectors at the LHC by Vector Boson Fusion and Drell-Yan annihilation has been studied in this framework. The effective Lagrangian description of the interactions of these vectors, among themselves or with the Standard Model gauge bosons, has several free parameters and gives rise in general to scattering amplitudes with bad asymptotic behaviour. In order to avoid the saturation of perturbative unitarity, relations among the different parameters should exist. These relations have been used to constrain the parameter space. The connection between a composite vector and a gauge vector of a spontaneously broken gauge symmetry has been discussed. For a reasonable effective theory approach one can only accept relatively small deviations of the parameters from those corresponding to a good asymptotic behavior of the various physical amplitudes, since large deviations quickly lower the cutoff to unacceptably small values. The total cross sections at the LHC for the vector pair production by Vector Boson Fusion and Drell-Yan annihilation are of order of few fb. The numbers of same sign di-lepton and trilepton events at the LHC with an integrated luminosity of 100 fb^{-1} are of the order of 10.

In the second part of the thesis a Higgs-like scalar h and a vector V^a, triplet under the custodial $SU(2)_{L+R}$, have been considered in the framework of a $SU(2)_L \times SU(2)_R/SU(2)_{L+R}$ Effective Lagrangian which describes the interactions of these states. In order to have a reasonable Effective Lagrangian description of these particles, the interactions of the V^a among themselves and with the electroweak gauge bosons have been restricted to those resulting from a $SU(2)_L \times SU(2)^N \times SU(2)_R$ gauge theory spontaneously broken to the diagonal $SU(2)_{L+R}$ subgroup by a generic non-linear sigma model. In this framework, the two body amplitudes for the scattering of the $W_L W_L$ initial state into the $W_L W_L, hh, V_L V_L, V_L h$ final states have been computed in terms of five couplings (a, b, d, g_V and g_K) and two masses (m_h and M_V). The relation of these amplitudes with those arising from an explicit $SU(2)_L \times SU(2)_C \times U(1)_Y$ gauge model spontaneously broken by Higgs multiplets has been clarified. The parameter space has been restricted by requiring a good high energy behaviour of the elastic $W_L W_L \to W_L W_L$ scattering amplitude. From a phenomenological point of view the associated production of a scalar and a heavy vector by Vector Boson Fusion and Drell-Yan annihilation has been studied. It has been found that for a vector with a mass between 500 GeV and 1 TeV and for $m_h = 180$ GeV, the main production mechanism at the LHC of a composite vector together with a composite scalar is by Drell-Yan annihilation. The order of magnitude of the cross sections is about 10 fb for a reasonable choice of the parameters. This value can also be strongly increased since it depends quadratically on the scalar-vector coupling d. The expected same sign di-lepton and tri-lepton events are of the order of $10 - 100$ for an integrated luminosity of 100 fb^{-1}. Further detailed studies, which are beyond the scope of this work will have to be made to assess the detectability at the LHC of composite vector pairs and composite vector-composite scalar final states above the Standard Model backgrounds.

The experimental investigation of all the processes that we have studied will only be possible at the LHC with its maximum energy and intensity. Before that, the single direct production of any composite state, if they exist at all, will have been discovered. Nevertheless, to unravel the structure of the underlying dynamics, the study of the processes considered in this thesis will probably be necessary. To this end the tools and the considerations developped here will hopefully prove useful.

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Appendix: A well behaved theory at all energies

Let us consider the following $SU(2)_L \times SU(2)_C \times U(1)_Y$ invariant non-linear sigma model Lagrangian:

$$\mathcal{L}^{\text{gauge}} = \mathcal{L}_\chi^{\text{gauge}} - \frac{1}{2g_C^2} \langle v_{\mu\nu} v^{\mu\nu} \rangle - \frac{1}{2g^2} \langle W_{\mu\nu} W^{\mu\nu} \rangle - \frac{1}{2g'^2} \langle B_{\mu\nu} B^{\mu\nu} \rangle - V\left(\Sigma_{YC}, \Sigma_{CL}\right) , \tag{4.1}$$

where

$$v_\mu = \frac{g_C}{2} v_\mu^a \tau^a \tag{4.2}$$

is the $SU(2)_C$-gauge vector and

$$\mathcal{L}_\chi^{\text{gauge}} = \frac{v^2}{2} \left\langle D_\mu \Sigma_{YC} \left(D^\mu \Sigma_{YC}\right)^\dagger \right\rangle + \frac{v^2}{2} \left\langle D_\mu \Sigma_{CL} \left(D^\mu \Sigma_{CL}\right)^\dagger \right\rangle \tag{4.3}$$

is the symmetry breaking Lagrangian and $V\left(\Sigma_{YC}, \Sigma_{CL}\right)$ is the scalar potential, which has the form

$$\begin{aligned} V\left(\Sigma_{YC}, \Sigma_{CL}\right) = {} & \frac{\mu^2 v^2}{2} \left\langle \Sigma_{YC} \Sigma_{YC}^\dagger \right\rangle + \frac{\mu^2 v^2}{2} \left\langle \Sigma_{CL} \Sigma_{CL}^\dagger \right\rangle - \frac{\lambda v^4}{4} \left(\left\langle \Sigma_{YC} \Sigma_{YC}^\dagger \right\rangle \right)^2 \\ & - \frac{\lambda v^4}{4} \left(\left\langle \Sigma_{CL} \Sigma_{CL}^\dagger \right\rangle \right)^2 - \kappa v^4 \left\langle \Sigma_{YC} \Sigma_{CL}^\dagger \Sigma_{CL} \Sigma_{YC}^\dagger \right\rangle . \end{aligned} \tag{4.4}$$

To ensure the correct normalization for the Goldstone bosons kinetic terms, Σ_{YC} and Σ_{CL} are defined as:

$$\Sigma_{YC} = \left(1 + \frac{h+H}{2v}\right) U_{YC} , \qquad U_{YC} = \exp\left[\frac{i}{2v}\left(\pi + \sigma\right)\right] , \tag{4.5}$$

$$\Sigma_{CL} = \left(1 + \frac{h-H}{2v}\right) U_{CL} , \qquad U_{CL} = \exp\left[\frac{i}{2v}\left(\pi - \sigma\right)\right] , \tag{4.6}$$

where $\pi = \pi^a \tau^a$ and $\sigma = \sigma^a \tau^a$, being π^a and σ^a the Goldstone bosons respectively associated with the EW gauge bosons W_μ^a and with the heavy vectors v_μ^a and τ^a the usual Pauli matrices. Furthermore h and H are the physical L-R-parity even and odd scalars respectively and are assumed to have the same VEV v and to have the following masses

$$m_h^2 = 4v^2\left(\lambda + \kappa\right) , \qquad m_H^2 = 4v^2\left(\lambda - \kappa\right) . \tag{4.7}$$

The two Higgs doublets realize the spontaneous breaking of the $SU(2)_L \times SU(2)_C \times U(1)_Y$ local

symmetry to $U(1)_{\rm em}$, while the global group $G = SU(2)_L \times SU(2)_C \times SU(2)_R$ is broken to the diagonal subgroup $H = SU(2)_{L+C+R}$. The covariant derivatives appearing in (4.3) are given by

$$D_\mu U_{YC} = \partial_\mu U_{YC} - iB_\mu U_{YC} + iU_{YC} v_\mu \,, \qquad D_\mu U_{CL} = \partial_\mu U_{CL} - iv_\mu U_{CL} + iU_{CL} W_\mu \,. \quad (4.8)$$

The U fields can be written as $U_{YC} = \sigma_Y \sigma_C^\dagger$ and $U_{CL} = \sigma_C \sigma_L^\dagger$ where the $\sigma_{L,C,Y}$ are elements of $SU\left(2\right)_{L,C,R}/H$ respectively[7]. These σ_I with $I = L, C, Y$ transform under the full $SU\left(2\right)_L \times SU\left(2\right)_C \times U\left(1\right)_Y$ as $\sigma_I \to g_I \sigma_I h^\dagger$. By applying the gauge transformation

$$v_\mu^I \to \sigma_I^\dagger v_\mu^I \sigma_I + i\sigma_I^\dagger \partial_\mu \sigma_I = \Omega_\mu^I, \qquad U_{IJ} \to \sigma_I^\dagger U_{IJ} \sigma_J = 1\,, \quad (4.9)$$

the symmetry breaking Lagrangian takes the form

$$\mathcal{L}_\chi^{\rm gauge} = \frac{v^2}{2}\left(1 + \frac{h+H}{2v}\right)^2 \left\langle \left(\Omega_\mu^Y - \Omega_\mu^C\right)^2 \right\rangle + \frac{v^2}{2}\left(1 + \frac{h-H}{2v}\right)^2 \left\langle \left(\Omega_\mu^L - \Omega_\mu^C\right)^2 \right\rangle \,. \quad (4.10)$$

After the gauge fixing $\sigma_Y = \sigma_L^\dagger = u^2 = U = e^{\frac{i\hat{\pi}}{v}}$ and $\sigma_C = 1$, which implies that $U_{YC} = U_{CL}$ (i.e. $\hat{\sigma} = 0$) corresponding to the unitary gauge in which we get rid of the Goldstone bosons associated with the heavy vectors v_μ^a, the Lagrangian of the previous expression becomes

$$\mathcal{L}_\chi^{\rm gauge} = v^2\left(1 + \frac{h^2+H^2}{4v^2} + \frac{h}{v}\right)\left(\left\langle \left(v_\mu - i\Gamma_\mu\right)^2 \right\rangle + \frac{1}{4}\left\langle u_\mu u^\mu \right\rangle\right) - \frac{1}{2}\left(2vH + hH\right)\left\langle u^\mu \left(v_\mu - i\Gamma_\mu\right)\right\rangle \,, \quad (4.11)$$

where

$$u_\mu = \Omega_\mu^Y - \Omega_\mu^L = iu^\dagger D_\mu U u^\dagger, \qquad \Gamma_\mu = \frac{1}{2i}\left(\Omega_\mu^Y + \Omega_\mu^L\right) = \frac{1}{2}\left[u^\dagger\left(\partial_\mu - iB_\mu\right)u + u\left(\partial_\mu - iW_\mu\right)u^\dagger\right]. \quad (4.12)$$

Now by setting

$$v_\mu = V_\mu + i\Gamma_\mu \,, \quad (4.13)$$

by using the identity [29]

$$v_{\mu\nu} = V_{\mu\nu} - i\left[V_\mu, V_\nu\right] + \frac{i}{4}\left[u_\mu, u_\nu\right] + \frac{1}{2} f_{\mu\nu}^+ \,, \quad (4.14)$$

where $f_{\mu\nu}^+ = uW_{\mu\nu}u^\dagger + u^\dagger B_{\mu\nu} u$, and by redefining $V_\mu \to \frac{g_C}{\sqrt{2}} V_\mu$, we obtain the following effective Lagrangian

$$\mathcal{L}^{\rm gauge} = \mathcal{L}_{h=H=0} + \mathcal{L}_{h,H} \,, \quad (4.15)$$

[7]Remember that only the generator T^3 of $SU\left(2\right)_R$ is gauged.

where $\mathcal{L}_{h=H=0}$ and $\mathcal{L}_{h,H}$ are given by:

$$
\begin{aligned}
\mathcal{L}_{h=H=0} = & -\frac{1}{2g^2}\left\langle W_{\mu\nu}W^{\mu\nu}\right\rangle - \frac{1}{2g'^2}\left\langle B_{\mu\nu}B^{\mu\nu}\right\rangle - \frac{1}{4}\left\langle V_{\mu\nu}V^{\mu\nu}\right\rangle + \frac{v^2}{4}\left\langle D_\mu U\left(D^\mu U\right)^\dagger\right\rangle + \frac{M_V^2}{2}\left\langle V_\mu V^\mu\right\rangle \\
& + \frac{ig_C}{2\sqrt{2}}\left\langle V_{\mu\nu}\left[V^\mu, V^\nu\right]\right\rangle - \frac{g_C^2}{8}\left\langle \left[V_\mu, V_\nu\right]\left[V^\mu, V^\nu\right]\right\rangle - \frac{i}{4\sqrt{2}g_C}\left\langle V_{\mu\nu}\left[u^\mu, u^\nu\right]\right\rangle \\
& - \frac{1}{8}\left\langle \left[V_\mu, V_\nu\right]\left[u^\mu, u^\nu\right]\right\rangle - \frac{1}{2\sqrt{2}g_C}\left\langle V_{\mu\nu}f^{+\mu\nu}\right\rangle + \frac{i}{4}\left\langle \left[V^\mu, V^\nu\right]f^{+\mu\nu}\right\rangle \\
& + \frac{1}{32g_C^2}\left\langle \left[u_\mu, u_\nu\right]\left[u^\mu, u^\nu\right]\right\rangle - \frac{1}{8g_C^2}\left\langle f^+_{\mu\nu}f^{+\mu\nu}\right\rangle - \frac{i}{8g_C^2}\left\langle \left[u^\mu, u^\nu\right]f^{+\mu\nu}\right\rangle , \qquad (4.16)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{h,H} = & \, v^2\left(\frac{h^2+H^2}{4v^2} + \frac{h}{v}\right)\left(\frac{g_C^2}{2}\left\langle V_\mu V^\mu\right\rangle + \frac{1}{4}\left\langle D_\mu U\left(D^\mu U\right)^\dagger\right\rangle\right) \\
& - \frac{g_C}{2\sqrt{2}}\left(2vH + hH\right)\left\langle u^\mu V_\mu\right\rangle + \frac{1}{2}\left[\left(\partial_\mu h\right)^2 + \left(\partial_\mu H\right)^2\right] - V\left(h, H\right) , \qquad (4.17)
\end{aligned}
$$

and with the potential $V\left(h,H\right)$ given by

$$
\begin{aligned}
V\left(h,H\right) = & -\mu^2 v^2\left(1+\frac{h+H}{2v}\right)^2 - \mu^2 v^2\left(1+\frac{h-H}{2v}\right)^2 + 2\kappa v^4\left(1+\frac{h+H}{2v}\right)^2\left(1+\frac{h-H}{2v}\right)^2 \\
& + \lambda v^4\left(1+\frac{h+H}{2v}\right)^4 + \lambda v^4\left(1+\frac{h-H}{2v}\right)^4 . \qquad (4.18)
\end{aligned}
$$

By taking the mass of the L-R-parity odd H given in (4.7) infinitely large (so that it is decoupled from the theory), $\mathcal{L}^{\text{gauge}}$ coincides with $\mathcal{L}_{\text{eff}}$ in (3.1) up to operators irrelevant for the processes (3.7), only for the values of the parameters:

$$
\begin{aligned}
& g_V = \frac{1}{2g_C} = \frac{1}{g_K} = \frac{v}{2M_V}, \qquad f_V = 2g_V, \qquad M_V = g_C v = \frac{1}{2}g_K v = \frac{v}{2g_V} , \\
& a = \frac{1}{2}, \qquad b = \frac{1}{4}, \qquad d = 1, \qquad G_V = \frac{v}{2} . \qquad (4.19)
\end{aligned}
$$

This implies that when the relations (4.19) are satisfied, $\mathcal{L}_{\text{eff}}$ in (3.1) reduces to $\mathcal{L}^{\text{gauge}}$ in (4.15) in the limit $m_H \gg \Lambda$. Since the theory described by $\mathcal{L}^{\text{gauge}}$ is well behaved at all energies, the relations (4.19) allow to take under control the unitarity of the model under consideration.